# THE DRAG-REDUCTION OSCILLATING-WALL PROBLEM: NEW INSIGHT AFTER 20 YEARS

**Pierre Ricco†, Claudio Ottonelli⋆, Yosuke Hasegawa‡, Maurizio Quadrio•**

† The University of Sheffield

⋆ ONERA, Paris

‡ The University of Tokyo

• Politecnico di Milano

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### ACTIVE OPEN-LOOP TECHNIQUE

Energy input into system

Pre-determined forcing

Channel flow DNS ($Re_\tau = u_\tau h/\nu = 200$)

### SPANWISE WALL OSCILLATIONS

New approach: *Turbulent enstrophy*

*Transient evolution*

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$\tau_w$ is fixed in fully-developed conditions

**GAIN: $U_b$ increases**

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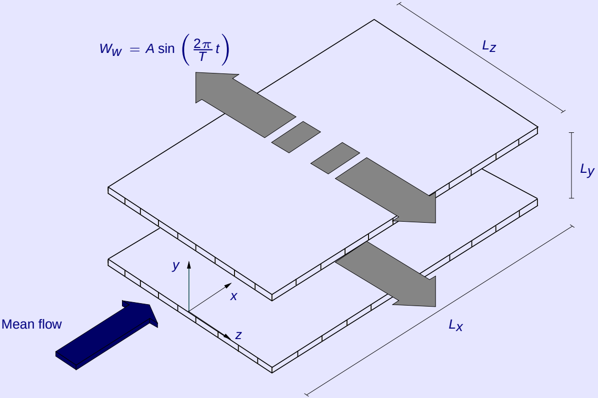
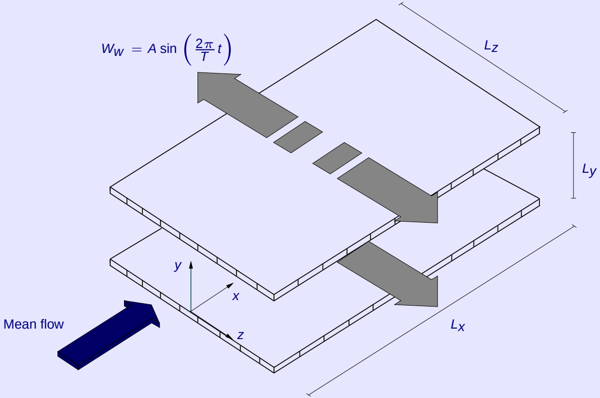



$$R = \frac{C_{f,r} - C_{f,o}}{C_{f,r}} = \frac{U_{b,o}^2 - U_{b,r}^2}{U_{b,o}^2}$$

**Why does the skin-friction coefficent decrease?**

$C_f = \tau_W / (1/2 \rho U_b^2)$ decreases $\longrightarrow$ study why $U_b$ increases

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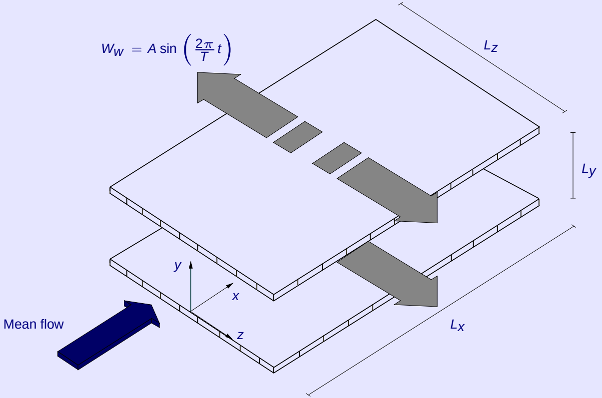


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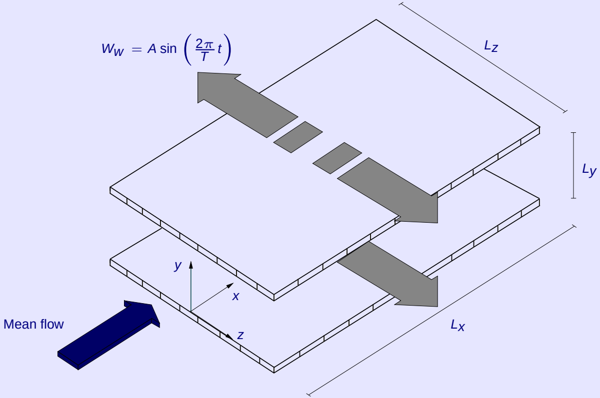


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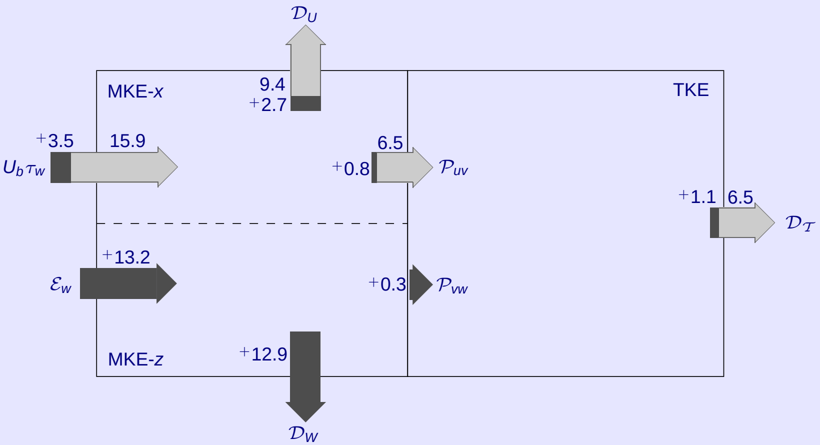
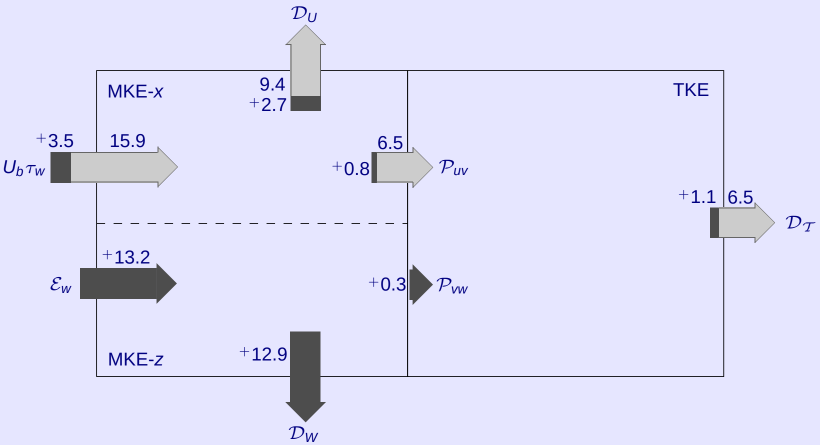



- Energy is fed through $P_x$ ($\rightarrow U_b\tau_w$) and wall motion ($\rightarrow \mathcal{E}_w$)
- Energy is dissipated through:
  - Mean-flow viscous effects ($\rightarrow \mathcal{D}_U, \mathcal{D}_W$)
  - Turbulent viscous effects ($\rightarrow \mathcal{D}_\mathcal{T}$)

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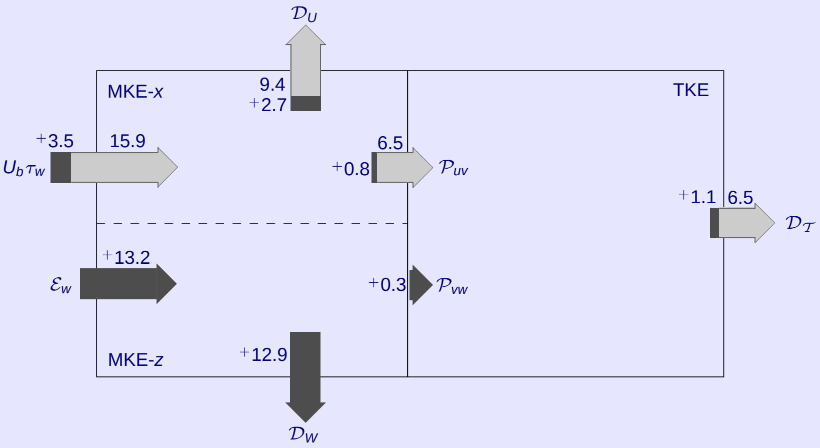


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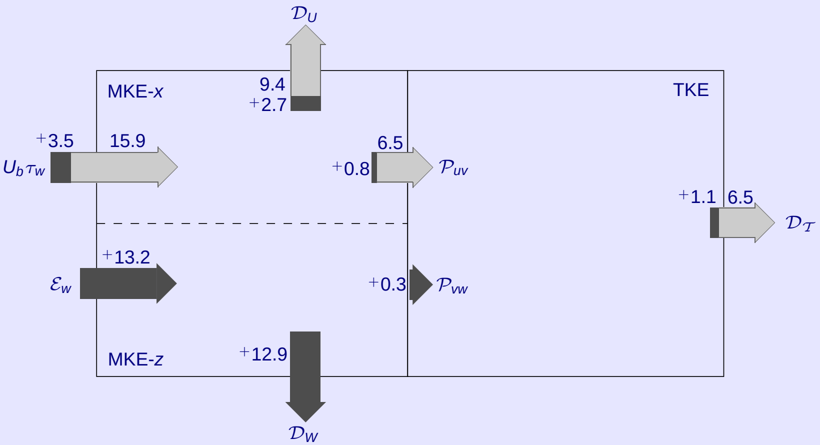


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### STILL TO BE ANSWERED

Why does TKE decrease?

Why does $U_b$ increase?

### DOES $W$ ACT ON TURBULENT DISSIPATION?

- Stokes-layer-type flow is generated by the wall oscillation
- Stokes layer's direct action on $\mathcal{D}_T = \int_V \widehat{\omega_i \omega_i} dV$
- Study the transport of turbulent enstrophy $\widehat{\omega_i \omega_i}$
- The term *enstrophy* was coined by G. Nickel and is from Greek $\sigma\tau\rho o\phi\acute{\eta}$
  $\longrightarrow$ *turn*

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$$+ \underbrace{\frac{1}{2}\frac{\partial^2 \widehat{\omega_i\omega_i}}{\partial y^2}}_{10} - \underbrace{\widehat{\frac{\partial \omega_i}{\partial x_j}\frac{\partial \omega_i}{\partial x_j}}}_{11}.$$

**Terms scaled in viscous units**

Stokes layer influences dynamics of turbulent enstrophy

Three terms: which is the dominating one?

$\rightarrow$ Let's look at the terms of the equation

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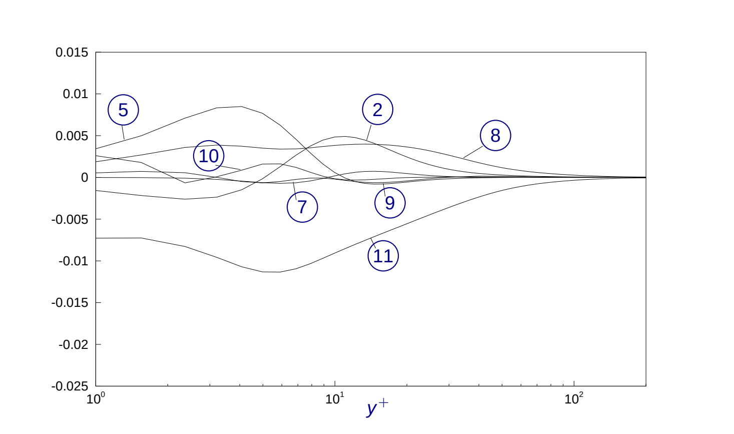
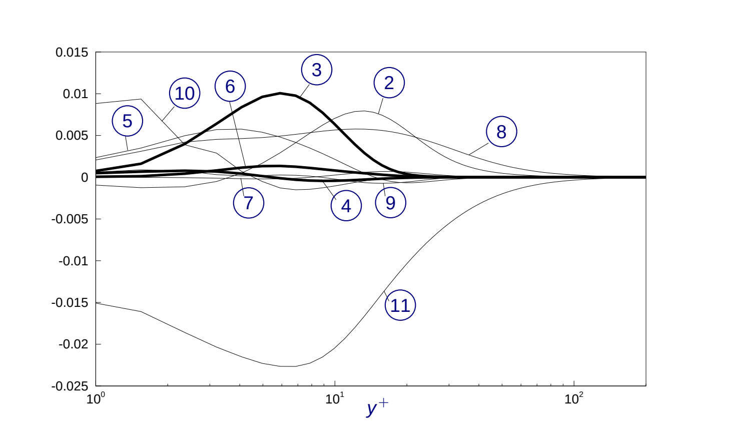
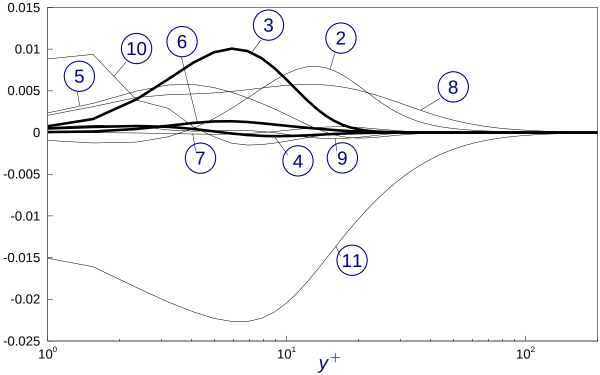

0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
10^0
10^1
10^2
y+
5
2
8
10
7
9
11




Term 3, $\widehat{\omega_z \omega_y} \partial \widehat{W} / \partial y \rightarrow$ turbulent enstrophy production is dominant

Turbulent dissipation of turbulent enstrophy increases

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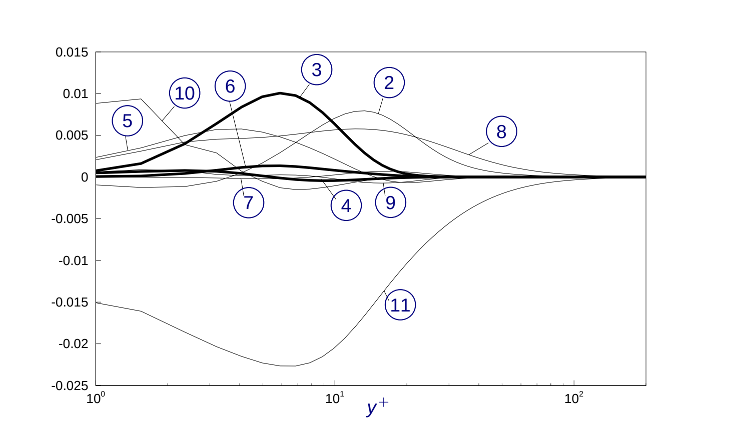


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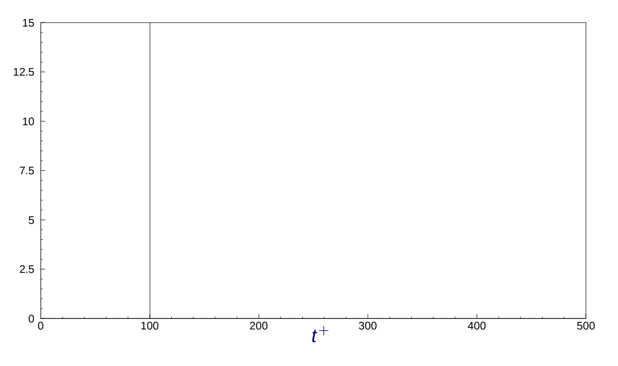
We have not answered questions on TKE and $U_b$, yet

Key: transient from start-up of wall motion

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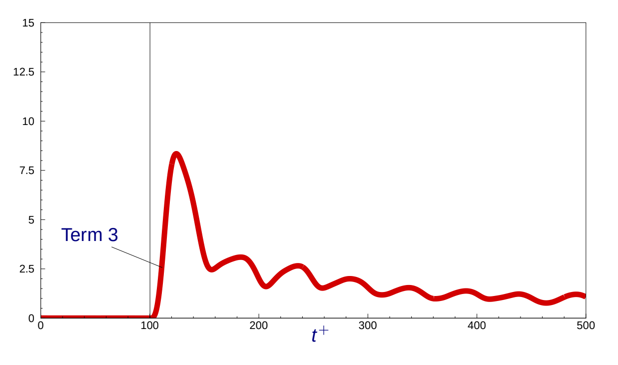


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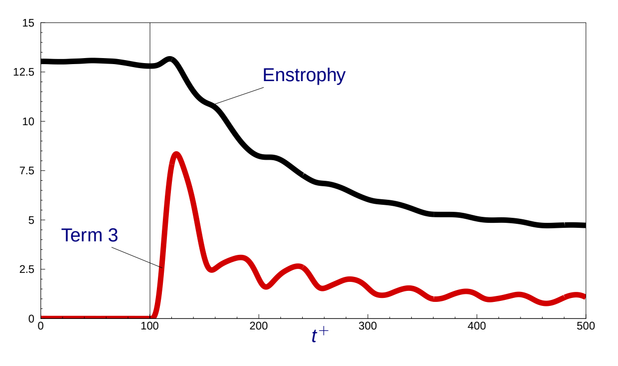


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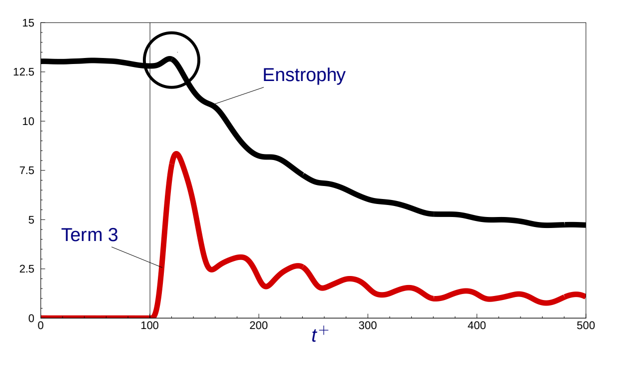


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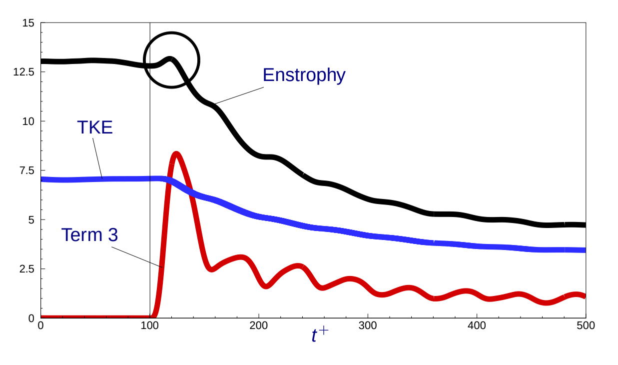



## USEFUL INFORMATION

**RED: term 3 increases abruptly**, then decreases

**BLACK: turbulent enstrophy increases**, then decreases

**BLUE: TKE decreases monotonically**

Initial state

Initial state

**Short**
$t^+ < 50$

$\overline{\omega_z \omega_y}^+ \frac{\partial \overline{W}^+}{\partial y^+} \uparrow \; \overline{\omega_i \omega_i}^+ \uparrow$

# DRAG REDUCTION MECHANISM

Initial state

**Short**

$t^+ < 50$

$\boxed{\mathcal{D}_{\mathcal{T}}^+ \uparrow}$

$\overline{\omega_z \omega_y}^+ \frac{\partial \overline{W}^+}{\partial y^+} \uparrow \quad \overline{\omega_i \omega_i}^+ \uparrow$

Initial state

**Short**

$t^+ < 50$

$\overline{\omega_z \omega_y}^+$ $\frac{\partial \overline{W}^+}{\partial y^+} \uparrow$ $\overline{\omega_i \omega_i}^+ \uparrow$

$\mathcal{D}_{\mathcal{T}}^+ \uparrow$

**Intermediate**

$50 < t^+ < 400$

TKE $\downarrow$ $\frac{\partial \overline{uv}^+}{\partial y^+} \downarrow$

Initial state

**Short**

$t^+ < 50$

$\overline{\omega_z \omega_y}^+ \ \frac{\partial \overline{W}^+}{\partial y^+} \uparrow \ \overline{\omega_i \omega_i}^+ \uparrow$

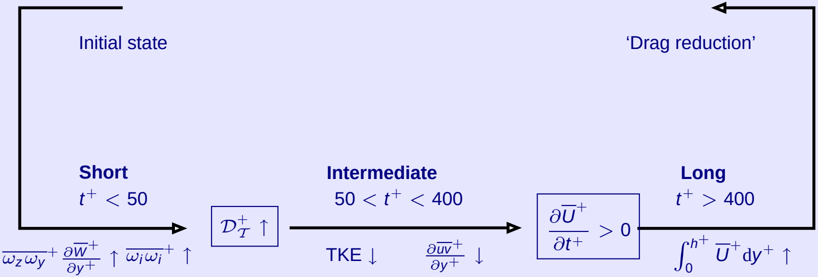
$\boxed{\mathcal{D}_{\mathcal{T}}^+ \uparrow}$

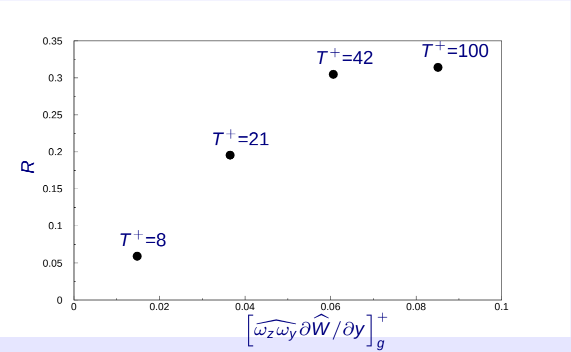
**Intermediate**

$50 < t^+ < 400$

TKE $\downarrow$ $\quad \frac{\partial \overline{uv}^+}{\partial y^+} \downarrow$

$\boxed{\frac{\partial \overline{U}^+}{\partial t^+} > 0}$


Initial state
'Drag reduction'
**Short**
$t^+ < 50$
**Intermediate**
$50 < t^+ < 400$
**Long**
$t^+ > 400$
$\mathcal{D}_{\mathcal{T}}^+ \uparrow$
$\frac{\partial \overline{U}^+}{\partial t^+} > 0$
$\overline{\omega_z \omega_y}^+ \frac{\partial \overline{W}^+}{\partial y^+} \uparrow$ $\overline{\omega_i \omega_i}^+ \uparrow$
TKE $\downarrow$
$\frac{\partial \overline{uv}^+}{\partial y^+} \downarrow$
$\int_0^{h^+} \overline{U}^+ \mathrm{d}y^+ \uparrow$



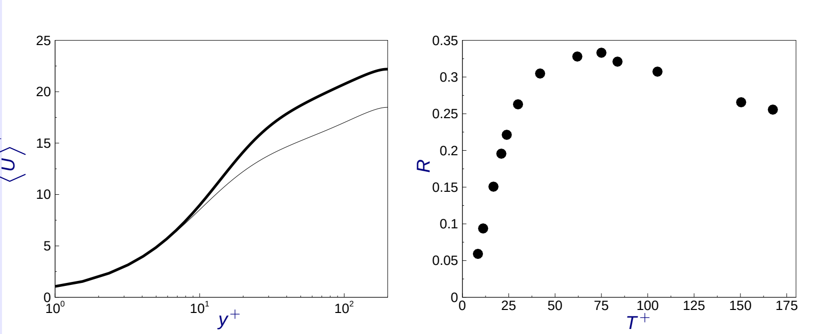

Drag reduction grows monotonically with global production term

This happens up to optimum period

## THANK YOU!

### Reference

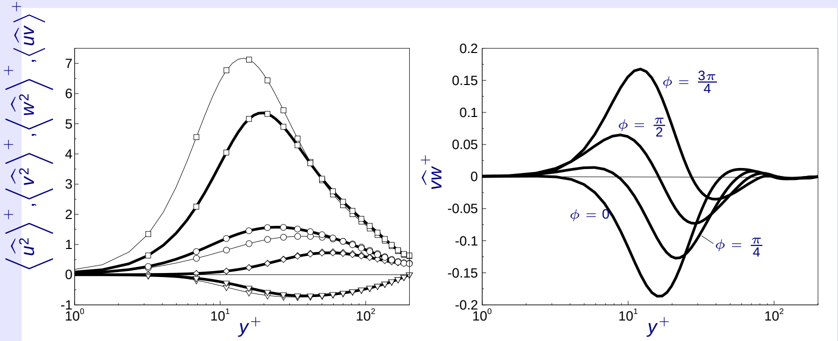
Ricco, P. Ottonelli, C. Hasegawa, Y. Quadrio, M.
Changes in turbulent dissipation in a channel flow with oscillating walls
*J. Fluid Mech.*, 700, 77-104, 2012.




Mean velocity increases in the bulk of the channel

Mean wall-shear stress is unchanged

Optimum period of oscillation $T^+ \approx 75$




Turbulence kinetic energy decreases

Streamwise velocity fluctuations are attenuated the most

New oscillatory Reynolds stress term $\widehat{vw}$ in created, $\langle \widehat{vw} \rangle = 0$

## GLOBAL MEAN KINETIC ENERGY EQUATION

$$\underbrace{U_b^+ \tau_w^+ + \left\langle A^+ \frac{\partial \widehat{W}^+}{\partial y^+}\bigg|_{y^+=0} \right\rangle}_{\mathcal{E}_W} = - \underbrace{\left[ \widehat{uv}^+ \frac{\partial \widehat{U}^+}{\partial y^+} \right]_g}_{\mathcal{P}_{uv}} - \underbrace{\left[ \widehat{vw}^+ \frac{\partial \widehat{W}^+}{\partial y^+} \right]_g}_{\mathcal{P}_{vw}} + \underbrace{\left[ \left( \frac{\partial \widehat{U}^+}{\partial y^+} \right)^2 \right]_g}_{\mathcal{D}_U} + \underbrace{\left[ \left( \frac{\partial \widehat{W}^+}{\partial y^+} \right)^2 \right]_g}_{\mathcal{D}_W}$$

## GLOBAL TURBULENT KINETIC ENERGY EQUATION

$$\underbrace{\left[ \widehat{uv}^+ \frac{\partial \widehat{U}^+}{\partial y^+} \right]_g}_{\mathcal{P}_{uv}} + \underbrace{\left[ \widehat{vw}^+ \frac{\partial \widehat{W}^+}{\partial y^+} \right]_g}_{\mathcal{P}_{vw}} + \left[ \overline{\frac{\partial u_i^+}{\partial x_j^+} \frac{\partial u_i^+}{\partial x_j^+}} \right]_g = 0$$

## TOTAL KINETIC ENERGY BALANCE

$$U_b^+ \tau_w^+ + \mathcal{E}_W^+ = \mathcal{D}_U^+ + \mathcal{D}_W^+ + \mathcal{D}_\mathcal{T}^+$$

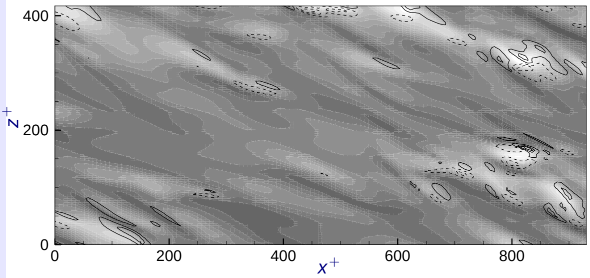
## TURBULENT DISSIPATION

$$\mathcal{D}_\mathcal{T}^+ = \left[ \widehat{\omega_i \omega_i} \right]_g^+$$

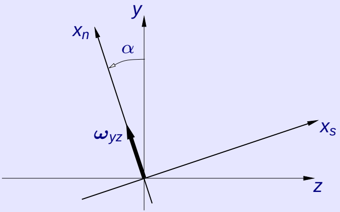
# Physical interpretation of $\widehat{\omega_z \omega_y} \partial \widehat{W} / \partial y$

- $\widehat{\omega_z \omega_y} \partial \widehat{W} / \partial y$ is key term leading to drag reduction
- $\widehat{\omega_z \omega_y} \partial \widehat{W} / \partial y \rightarrow \partial \widehat{W} / \partial y$ acts on $\widehat{\omega_z \omega_y}$
- $\widehat{\omega_z \omega_y} \approx \widehat{\frac{\partial u}{\partial y} \frac{\partial u}{\partial z}}$

  $\frac{\partial u}{\partial y} \rightarrow$ upward eruption of near-wall low-speed fluid
  $\frac{\partial u}{\partial z} \rightarrow$ lateral flanks of the low-speed streaks




$\frac{\partial u}{\partial y} \frac{\partial u}{\partial z}$ located at the sides of high-speed streaks




## SIMPLIFIED TURBULENT ENSTROPHY EQUATION

$$\frac{1}{2}\frac{\partial}{\partial t}\left(\omega_y^2+\omega_z^2\right)=\omega_z\omega_y G-\left(\frac{\partial \omega_y}{\partial y}\right)^2-\left(\frac{\partial \omega_z}{\partial y}\right)^2$$

Rotation of axis

$$\frac{1}{2}\frac{\partial \omega_n^2}{\partial t}=S_{nn}\omega_n^2-\left(\frac{\partial \omega_n}{\partial y}\right)^2$$

Integration by Charpit's method

$$\omega_n=\omega_{n,0}\underbrace{e^{\sin\alpha\cos\alpha G t}}_{\text{stretching}}\underbrace{e^{-\beta^2 t}e^{-\beta y}}_{\text{dissipation}},\ \beta=\frac{\partial\omega_n/\partial t}{\partial\omega_n/\partial y}\sim\frac{\lambda_y}{\lambda_t}$$