Asymptotic theory of laminar streaks and comparison with experiments

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OUTLINE

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- Klebanoff modes
  - Overview
  - Experimental and DNS results
  - Mathematical descriptions
- Open problem
  - Limit of theories
  - Objective
- Mathematical framework
- Results
- Comparison with experiments by Westin et al. JFM (1994)
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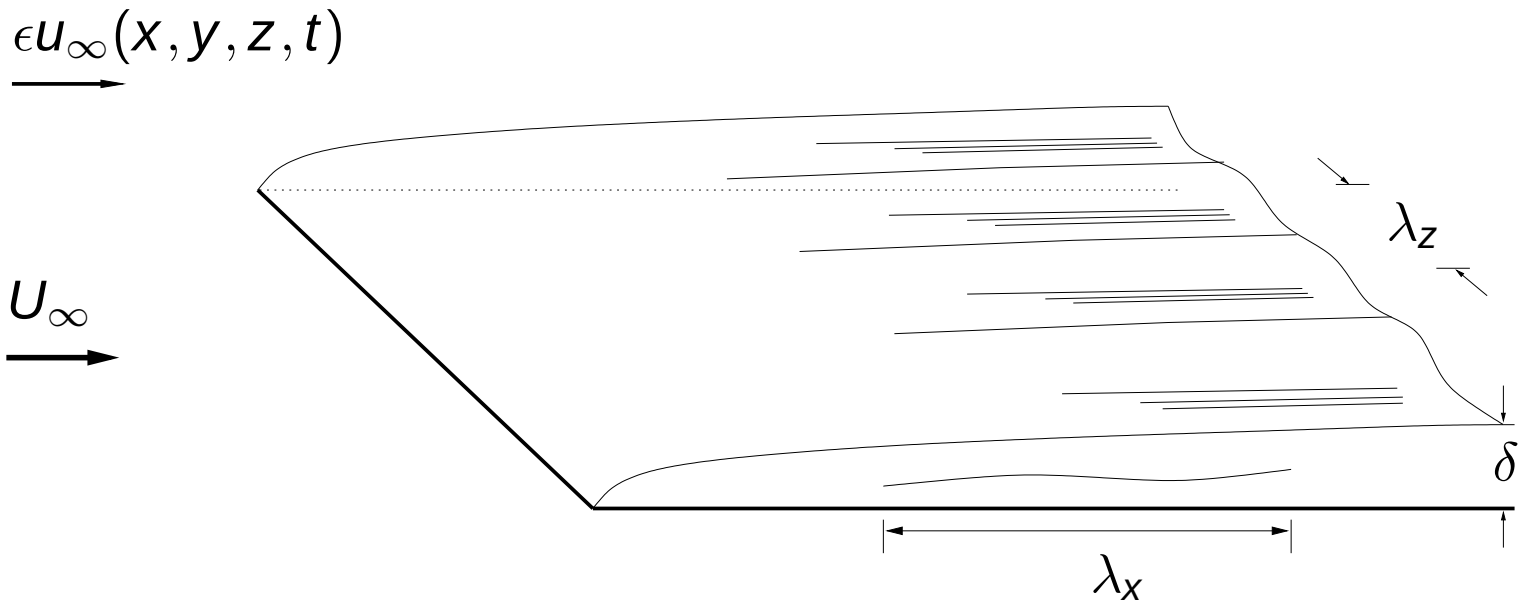
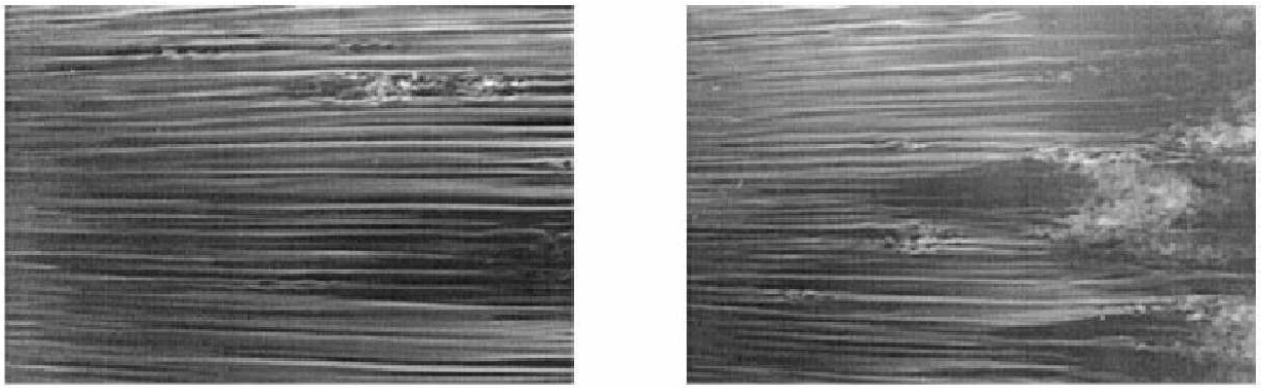
KLEBANOFF MODES: THE LAMINAR STREAKS



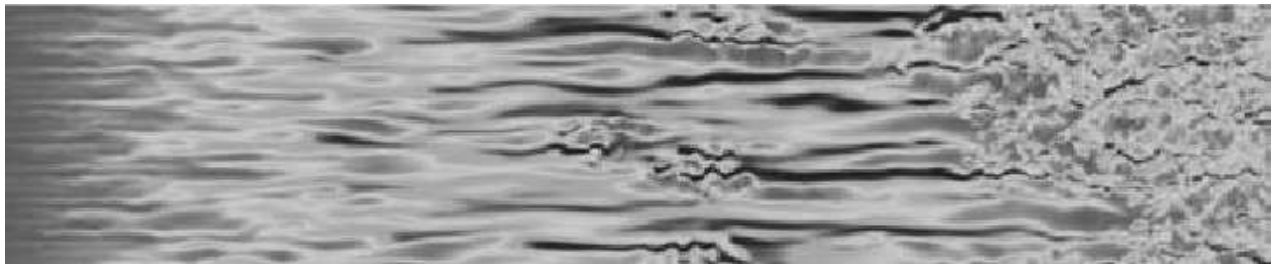
- Blasius layer perturbed by free-stream turbulence
- Filtering process:
  - Low frequency penetrate and amplify.
  - High frequency confine themselves in outer layer.
- Secondary instability: breakdown to turbulence.

FLOW VISUALIZATION

Wind tunnel: Matsubara & Alfredsson JFM (2001)



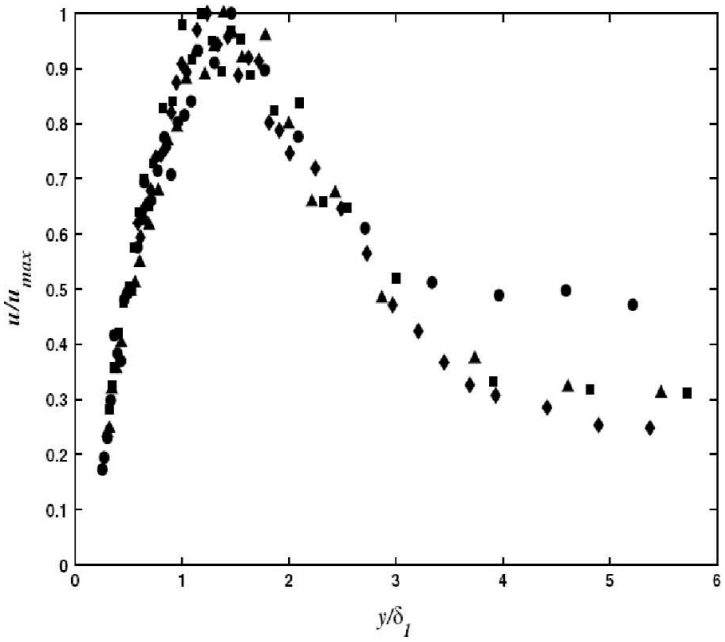
DNS: Brandt, Schlatter & Henningson JFM (2004)



- Streamwise elongated structures
- $u \gg v, w$

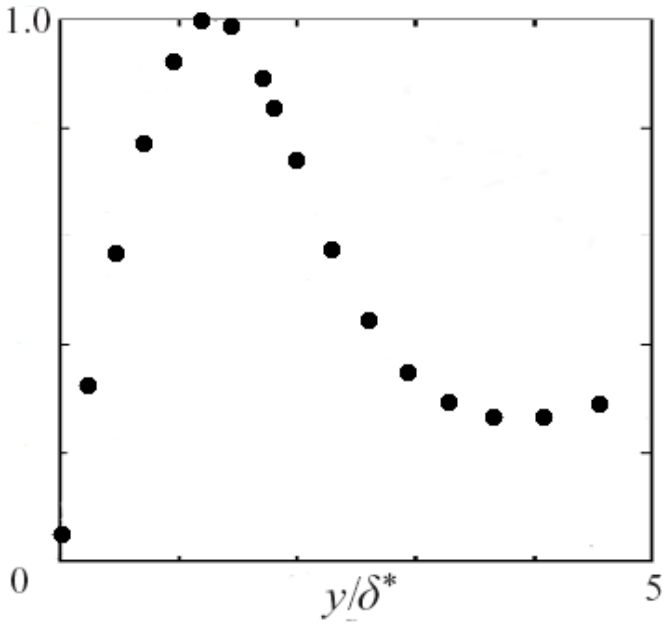
STREAK PROFILE

Wind tunnel



Hernon, Walsh & McEligot IJHFF (2007)

DNS



Ovchinnikov, Choudhari & Piomelli JFM (2008)

- u_{rms} peak in middle of boundary layer
- $u_{rms} \sim x^{1/2}$
- Matching with free-stream perturbation

MATHEMATICAL DESCRIPTION

Three approaches

- **Stewartson mode** $u \sim \eta F''$ Taylor Proc. ICAM (1939); Stewartson JMP (1957)

  Small modulation of boundary layer thickness

- **Optimal growth** Andersson, Berggren & Henningson JFM (1999); Luchini JFM (2000)

  Maximization of energy growth downstream by iterative adjoint method

- **Goldstein theory** Leib, Wundrow, Goldstein JFM (1999) (LWG99)

  Asymptotic theory based on unsteady boundary region eqs. with free-stream forcing

Key observation

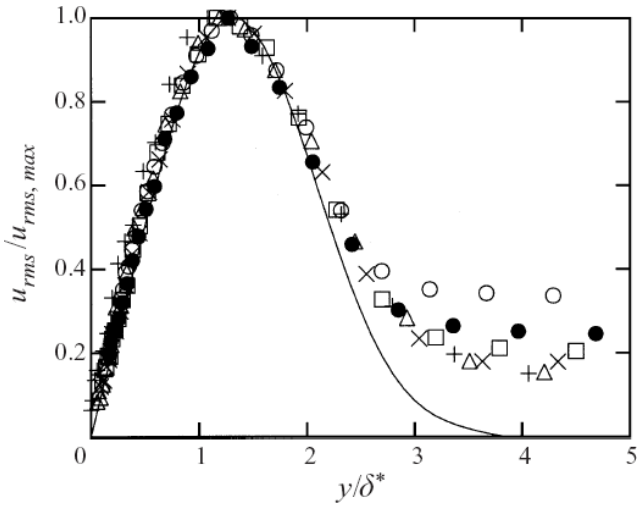
Good match with exp/DNS in **core** of boundary layer

Discrepancy in **outer layer** of boundary layer

Matsubara & Alfredsson JFM (2001) - wind tunnel

Choudhari AIAA Paper (1996); LWG99

Outer layer: $u \longrightarrow e^{ik_x(x-t)+ik_z z} \quad \lambda_z \gg \delta$



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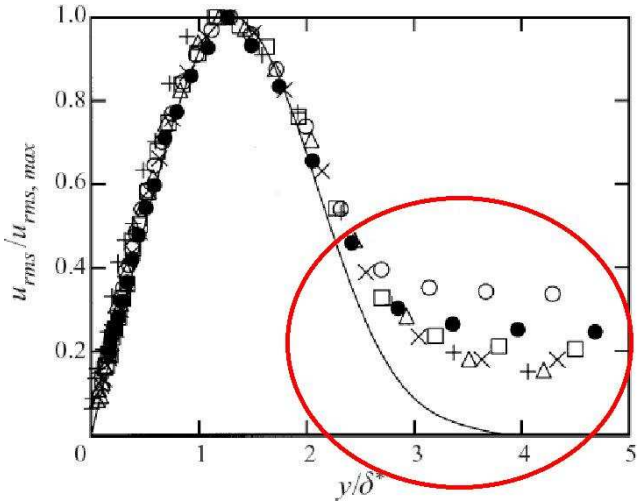

OBJECTIVE

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Explain physical mechanism in **outer layer**
Compare with experiments: Westin et al. JFM (1994)

Relevance

Jacobs & Durbin JFM (2001)
Low-frequency disturbances penetrate and distort the flow
High-frequency disturbances concentrate near free-stream
Outer layer: key to transition?

MATHEMATICAL FRAMEWORK

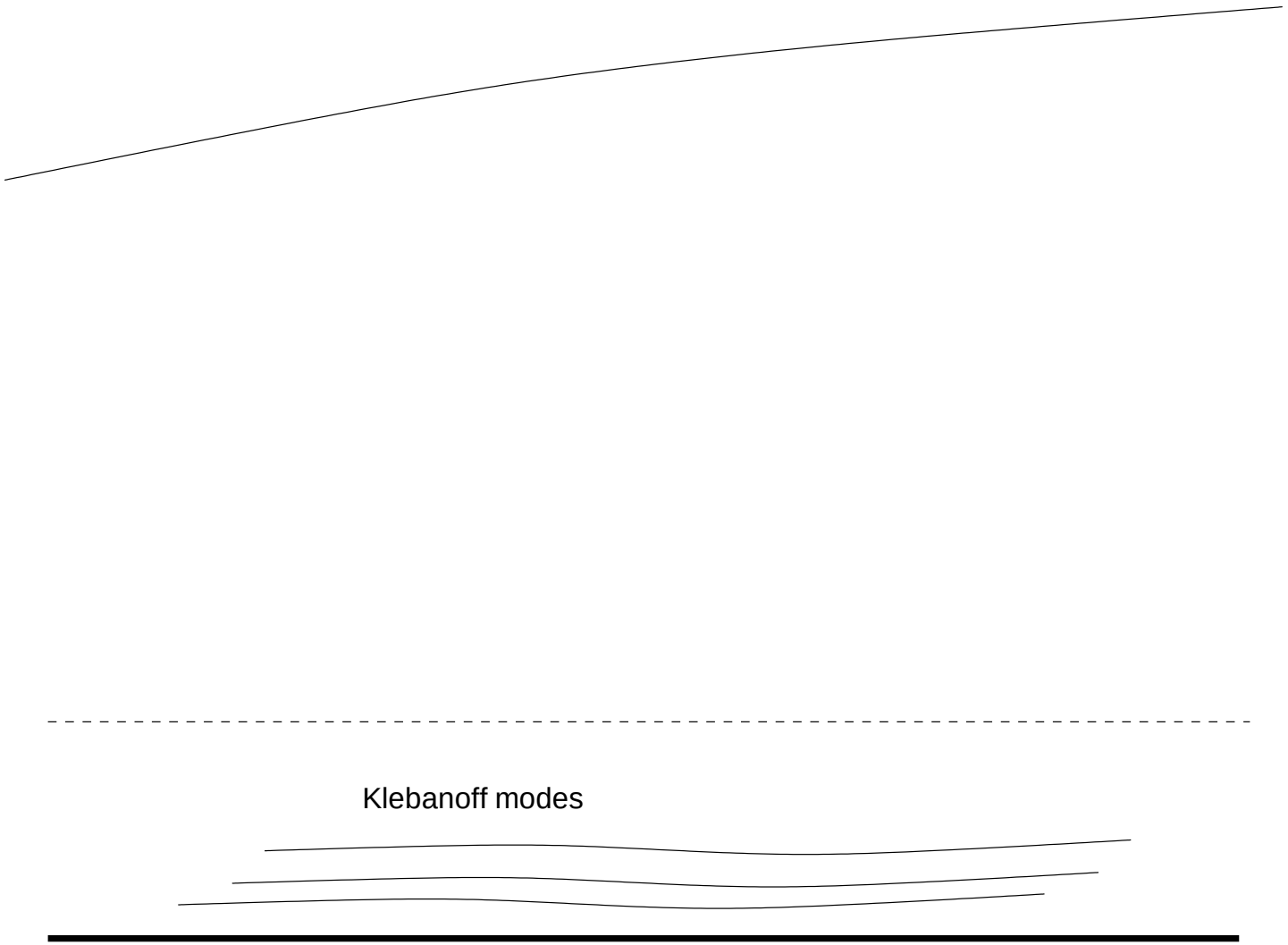
- Unsteady boundary region equations (LWG99)
  - Asymptotic limit of Navier-Stokes eqs. for $\omega \ll 1$
- Generation of *u* peak in core of boundary layer

  Free-stream *spanwise* *w* is the key

$$\frac{\partial w}{\partial \eta} + |\kappa|(2x)^{1/2} w \longrightarrow i\kappa_2 (2x)^{1/2} e^{i\left(x+\kappa_2 (2x)^{1/2}\eta\right)} e^{-\left(\kappa^2+\kappa_2^2\right)x}$$

  Free-stream *streamwise* *u* & *p* play no role
- Extend theory to reach our objective

OUTER LAYER MECHANISM



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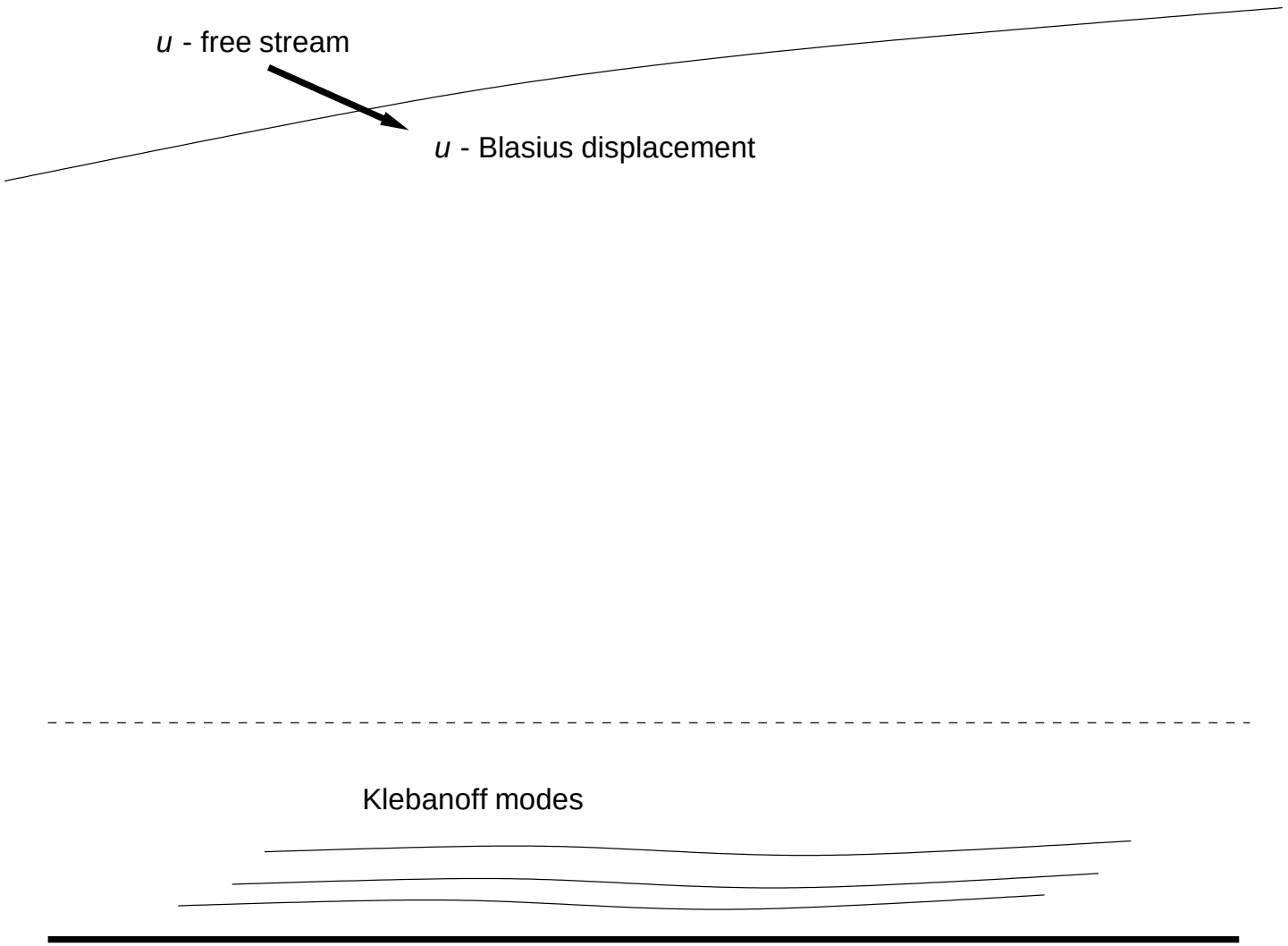

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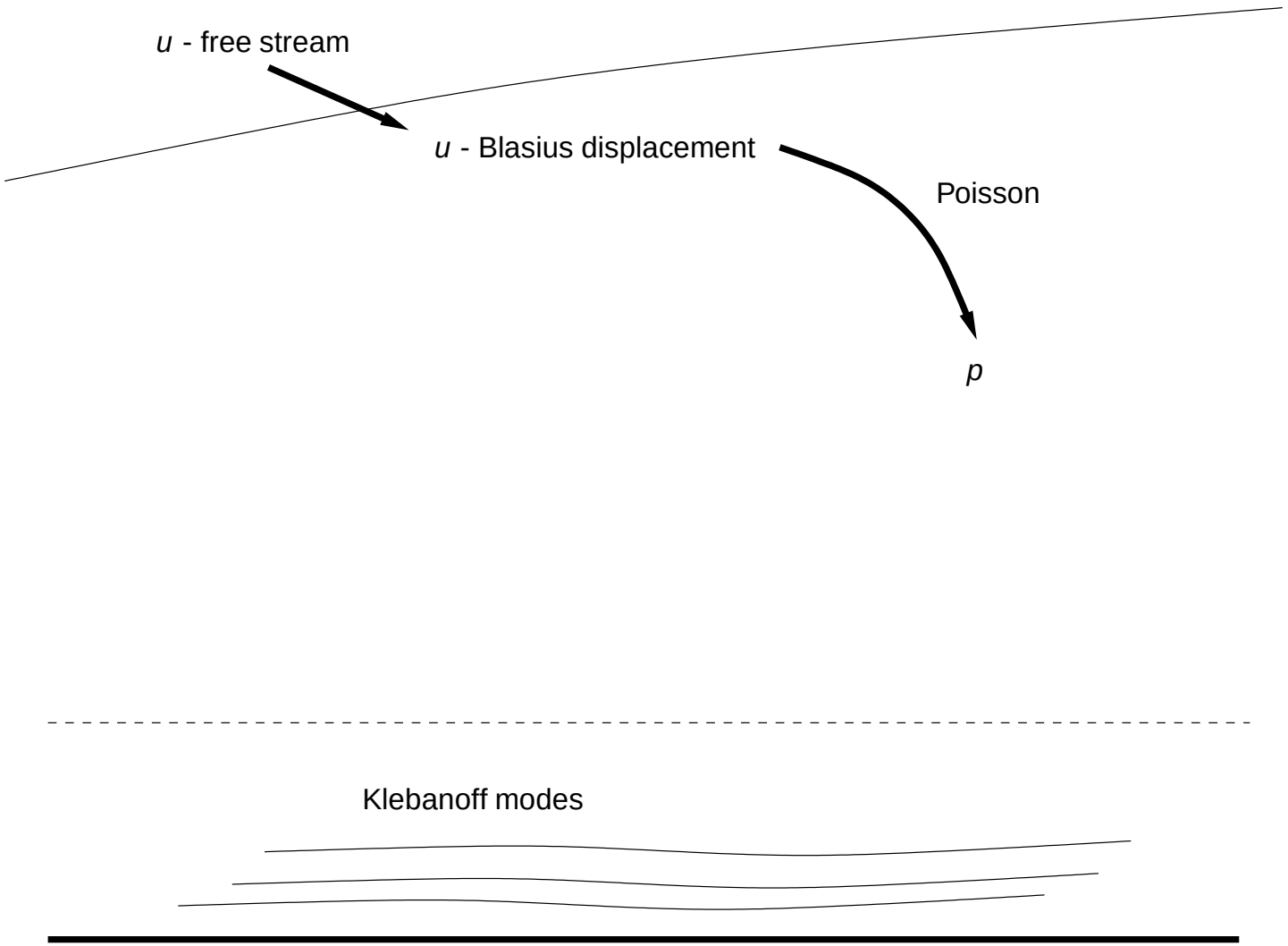

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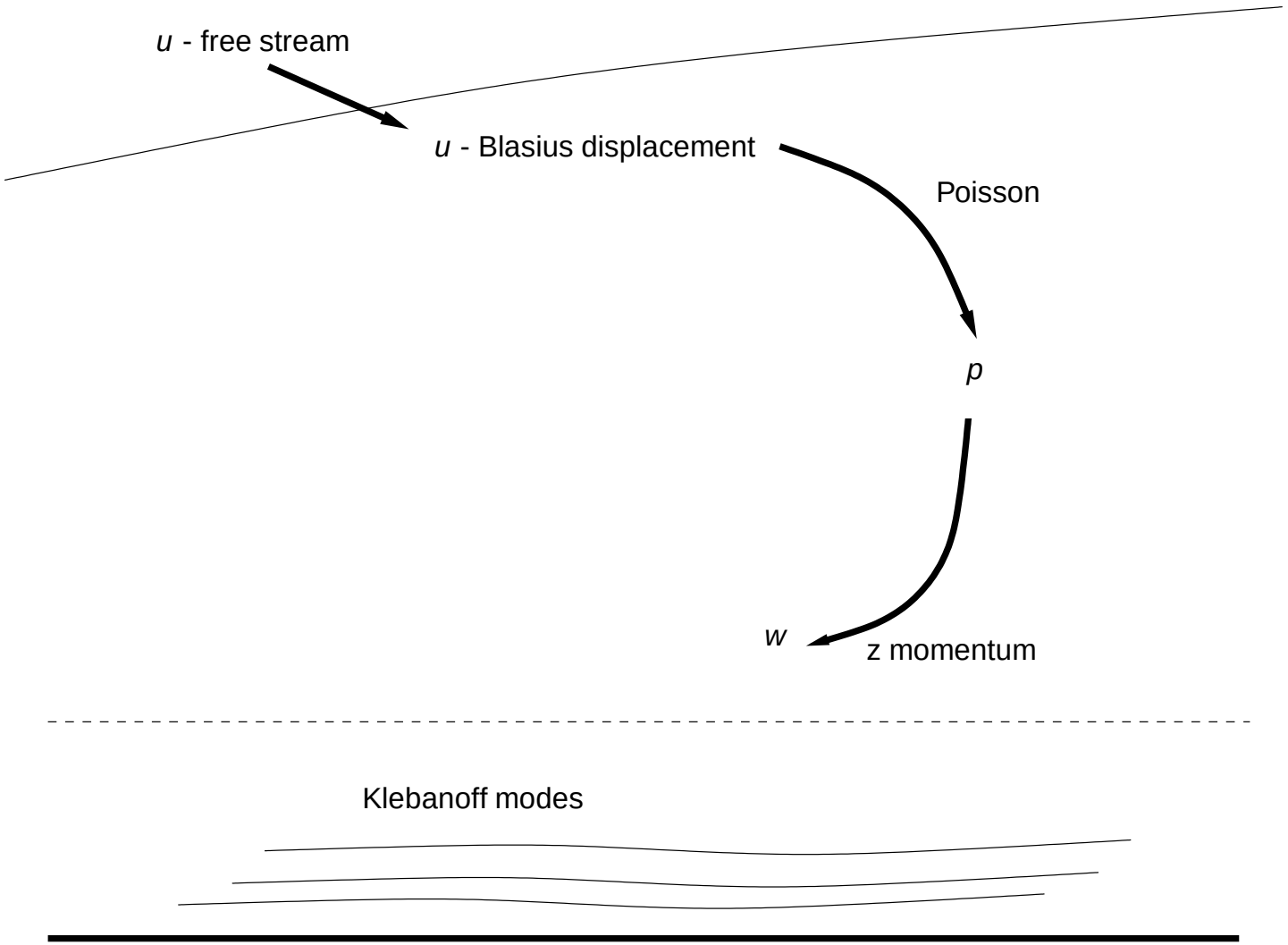

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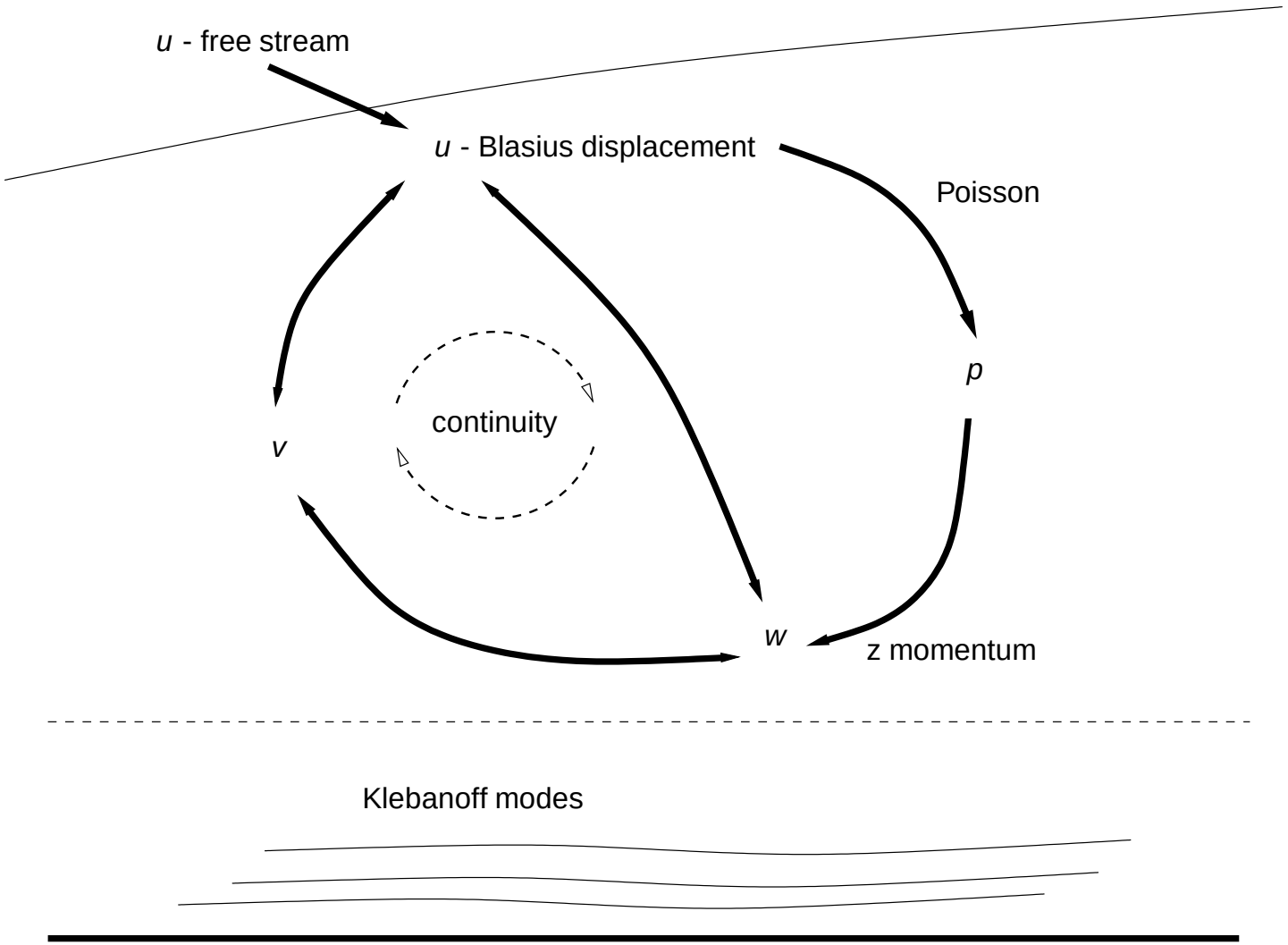

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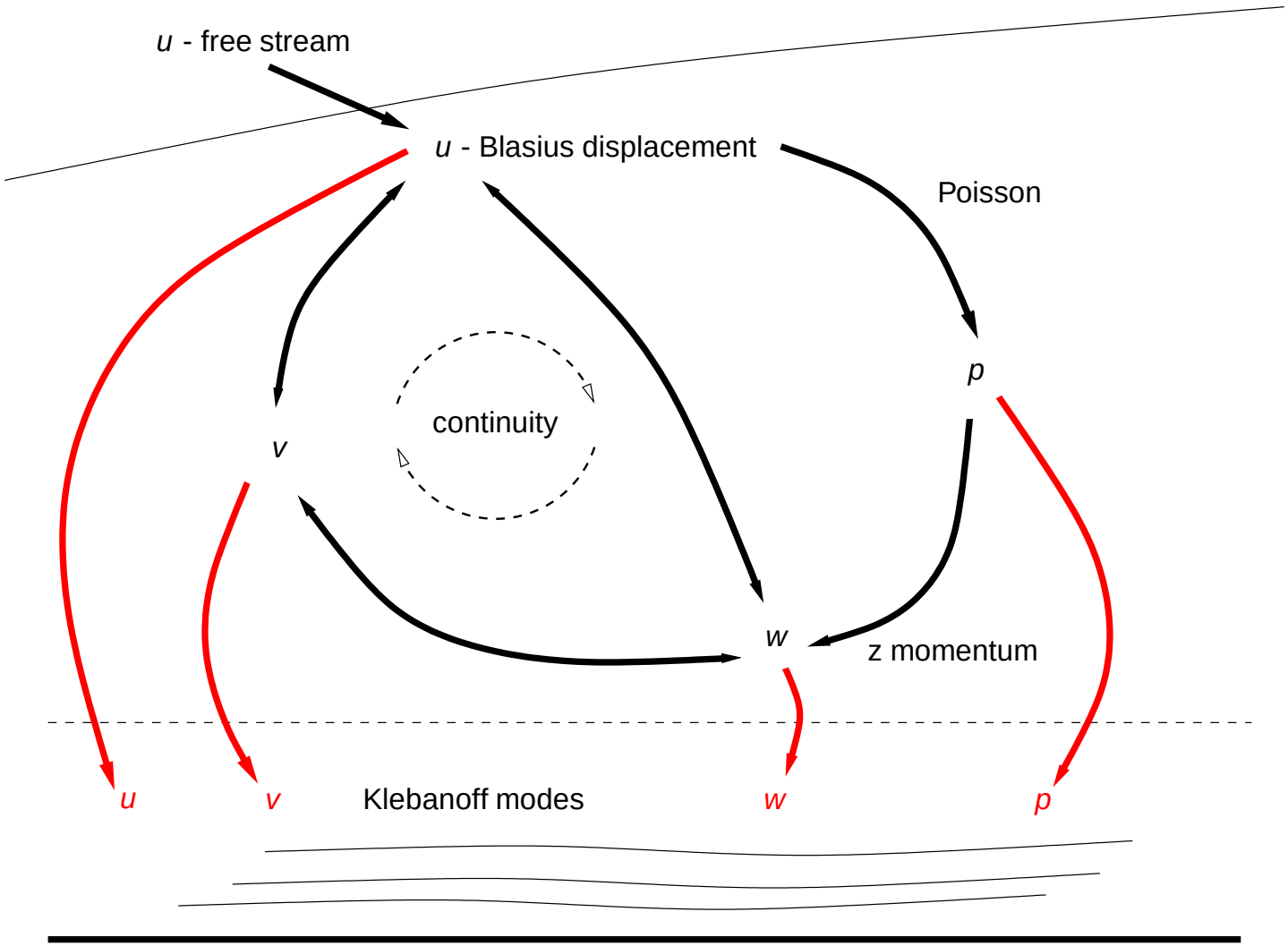


OUTER BOUNDARY CONDITIONS $\eta \to \infty$

$$u = \frac{e^{ix}}{\kappa_2 - i|\kappa|}\left(\kappa_2 e^{i\kappa_2(2x)^{1/2}\overline{\eta} - \left(\kappa^2+\kappa_2^2\right)x} - i|\kappa| e^{-|\kappa|(2x)^{1/2}\overline{\eta}}\right),$$

$$\frac{\partial v}{\partial \eta} + |\kappa|(2x)^{1/2} v \to \frac{i\beta\kappa^2 e^{ix-|\kappa|(2x)^{1/2}\overline{\eta}}}{(\kappa_2 - i|\kappa|)(2x)^{1/2}} + \left(\frac{i\kappa_2\beta\left(\kappa_2^2 - \kappa^2\right)}{(2x)^{1/2}\left(\kappa^2+\kappa_2^2\right)} - i + \kappa^2 + \kappa_2^2\right) e^{ix+i\kappa_2(2x)^{1/2}\overline{\eta} - \left(\kappa^2+\kappa_2^2\right)x}$$

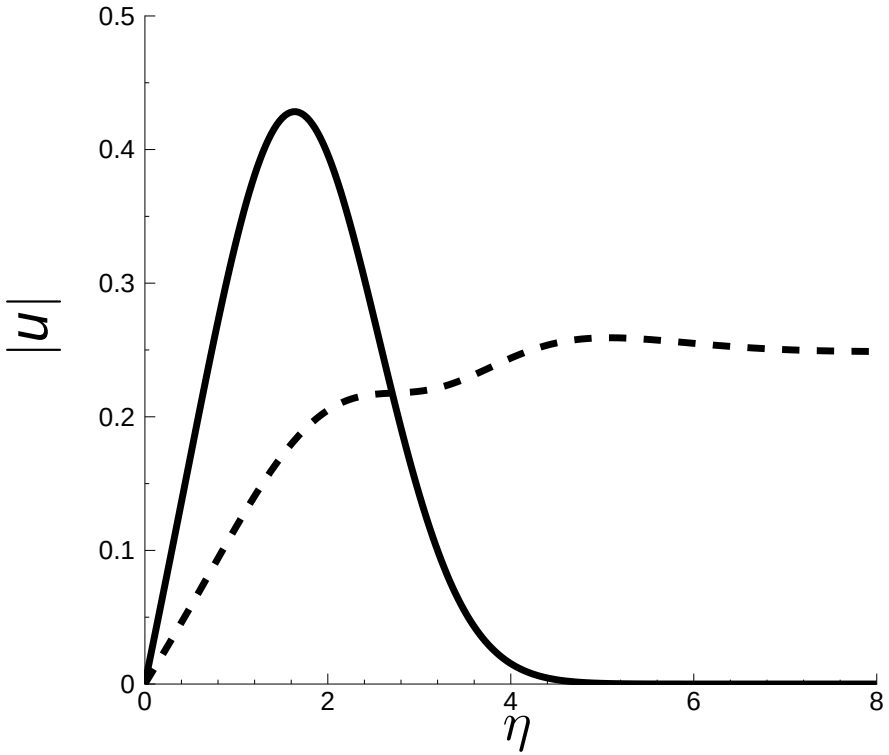
$$\frac{\partial w}{\partial \eta} + |\kappa|(2x)^{1/2} w \to \frac{i\beta|\kappa|^3 e^{ix-|\kappa|(2x)^{1/2}\overline{\eta}}}{\kappa_2 - i|\kappa|} - \frac{2\beta\kappa^2\kappa_2^2\, e^{ix+i\kappa_2(2x)^{1/2}\overline{\eta} - \left(\kappa^2+\kappa_2^2\right)x}}{\kappa^2+\kappa_2^2}$$

$$\frac{\partial p}{\partial \eta} + |\kappa|(2x)^{1/2} p \to -\frac{i\beta|\kappa| e^{ix-|\kappa|(2x)^{1/2}\overline{\eta}}}{2x(\kappa_2 - i|\kappa|)} + \frac{\beta\kappa_2^2\, e^{ix+i\kappa_2(2x)^{1/2}\overline{\eta} - \left(\kappa^2+\kappa_2^2\right)x}}{x\left(\kappa^2+\kappa_2^2\right)}$$

Mixed bc due to free-stream asymptotic matching through outer layer

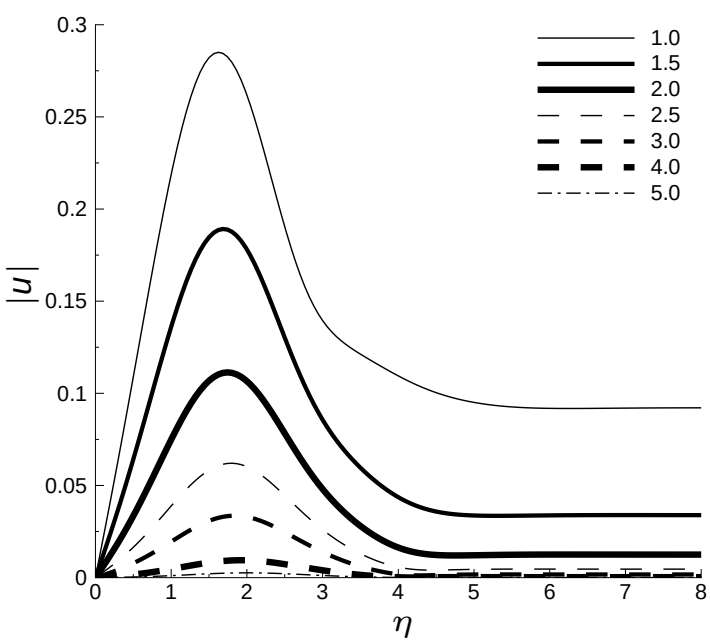
κ, κ_2: spanwise and wall-normal wavenumbers

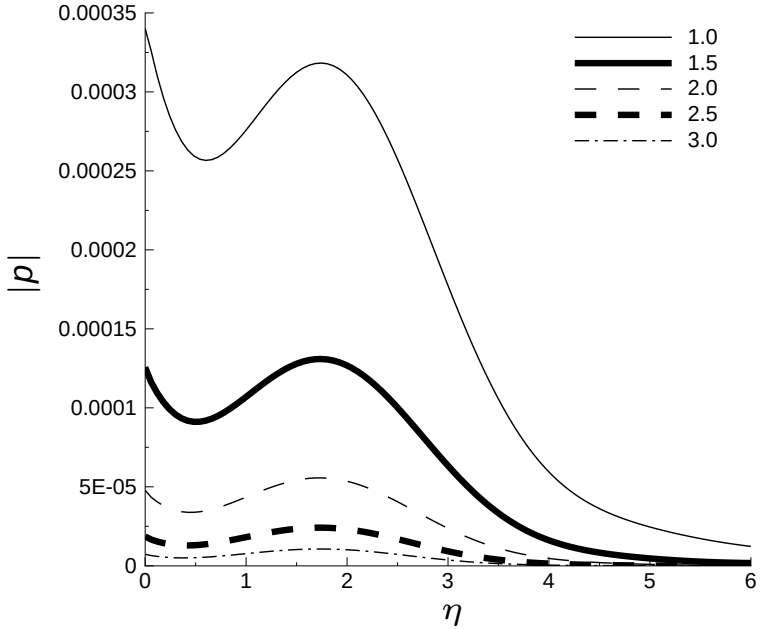
STREAMWISE VELOCITY PROFILE



- Solid line: driven by spanwise velocity
- Dashed line: outer-layer mechanism

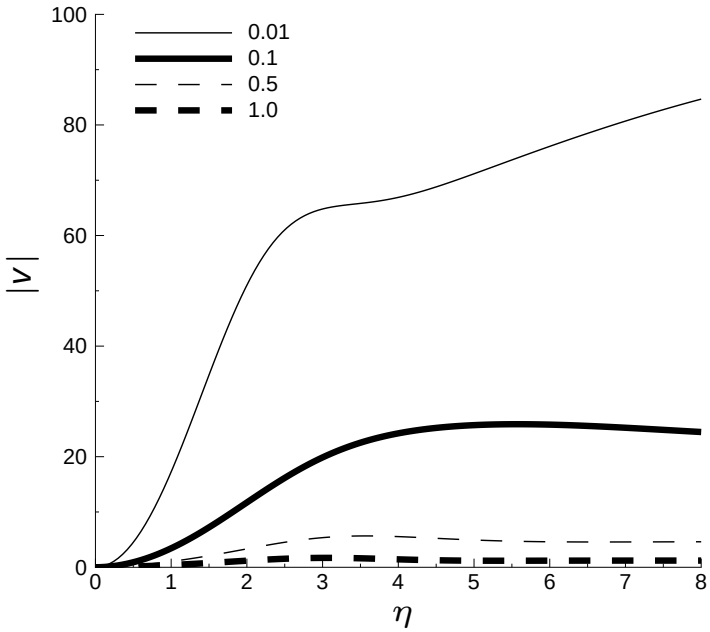
STREAMWISE VELOCITY & PRESSURE

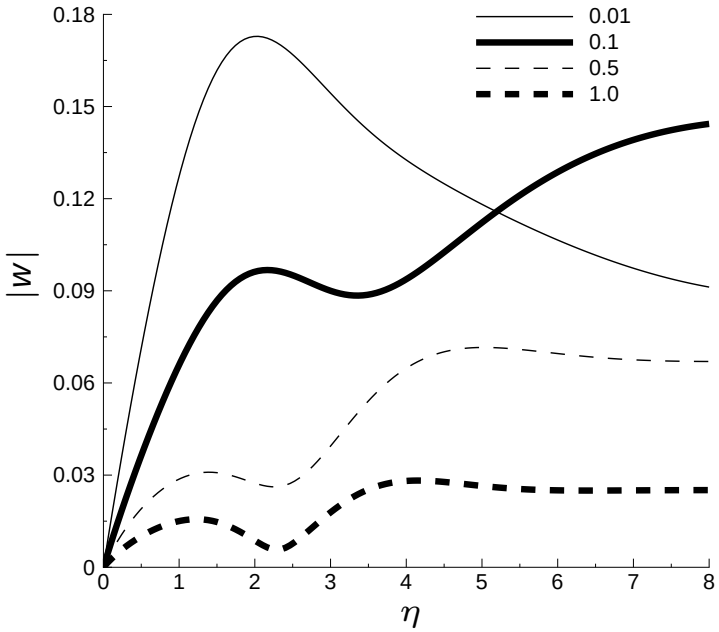




- Matching with free-stream fluctuation
- Pressure peak in middle of boundary layer

WALL-NORMAL & SPANWISE VELOCITY





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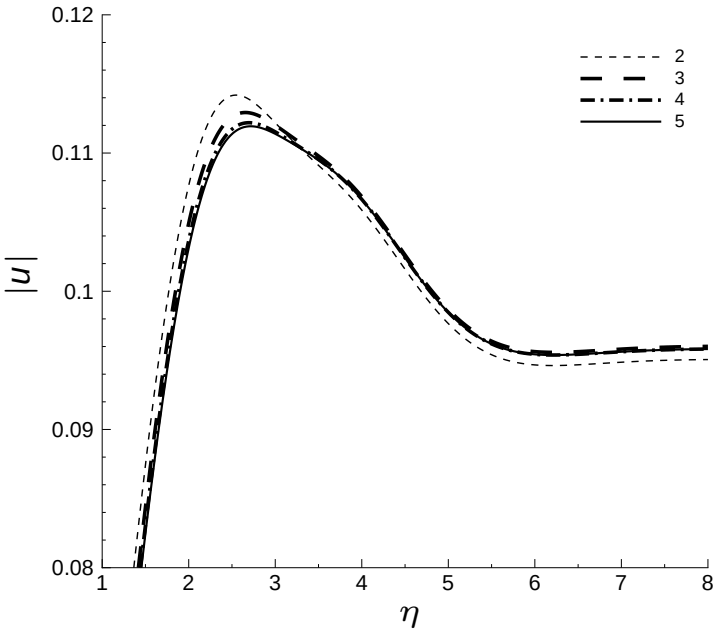
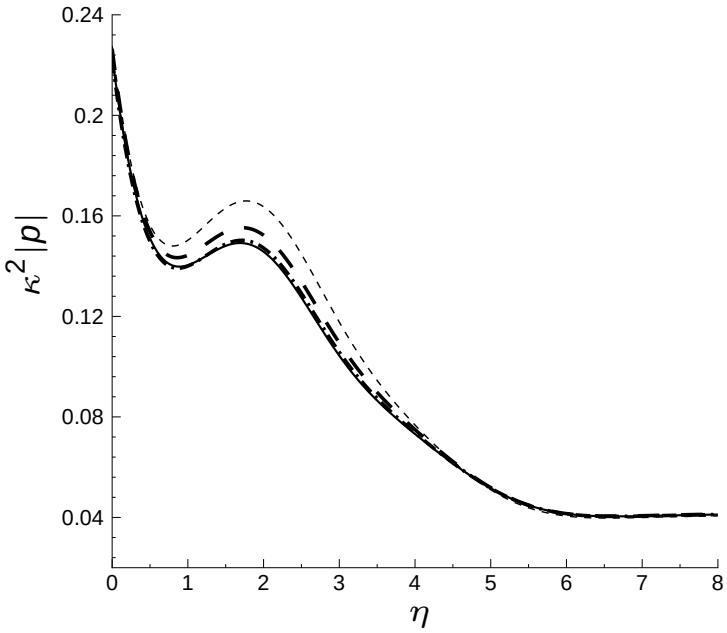
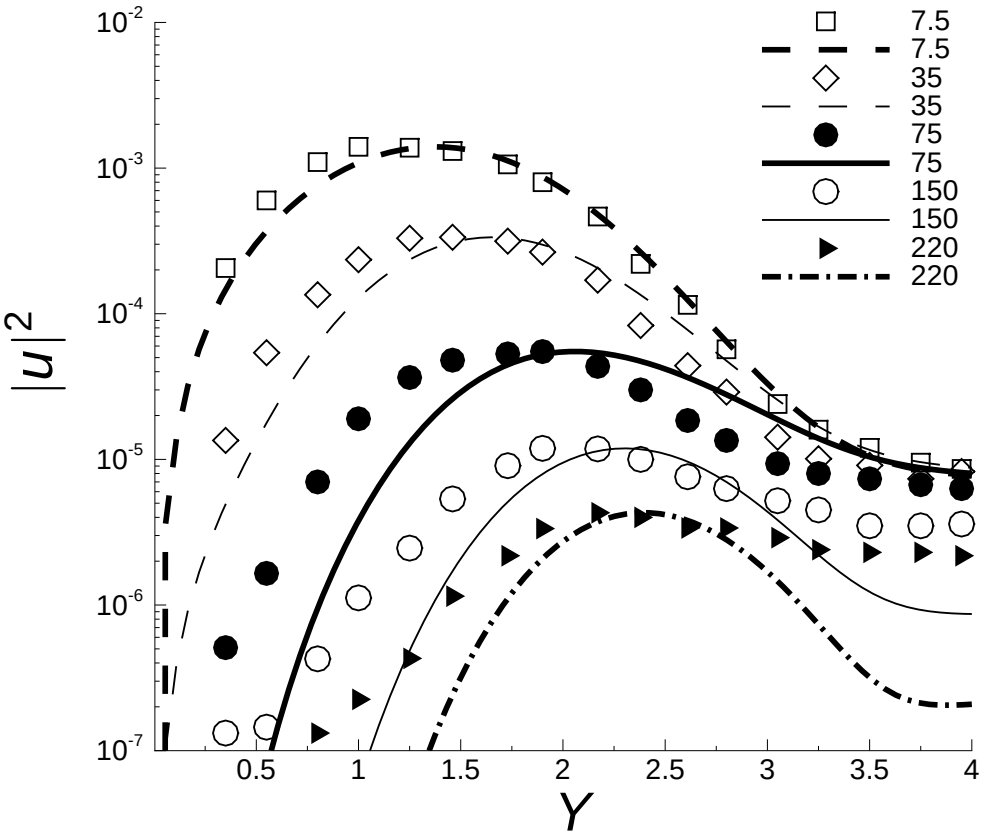
ASYMPTOTIC SCALING





- Limit $\kappa \gg 1$
- Steady boundary-region eqs

COMPARISON WITH EXPERIMENTS



- Westin et al. JFM (1994)
- Energy distribution at frequencies $\hat{F} = 10^6 \omega\nu/U_\infty^2$

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Motivation

- Generation and evolution of laminar streaks
- Outer-layer streak profile - comparison with experiments

Results

- Outer-layer physics: Poisson velocity-pressure interplay
- Description of laminar streaks across the whole boundary layer

Outlook

- Power of matched asymptotic analysis
- Key ingredient, together with experiments and DNS

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