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ASYMPTOTIC THEORY OF THE PRE-TRANSITIONAL LAMINAR STREAKS AND COMPARISON WITH EXPERIMENTS

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Summary The generation of the outer portion of the pre-transitional laminar streaks in the Blasius boundary layer is studied. A realistic streak profile is obtained and a good agreement with the experimental data by [4] is found.

INTRODUCTION

The present research work focusses on a mixed theoretical-numerical approach targeted at the modeling of the laminar streaks (or Klebanoff modes), namely the streamwise-elongated, low-frequency disturbances appearing in pre-transitional laminar boundary layers as a consequence of a medium-to-high level of free-stream turbulence. The streaks may be responsible for bypass transition, i.e. the breakdown to turbulence via some mechanism which is alternative to the classical instability route involving Tollmien-Schlichting waves. Experimental evidence indeed shows that, when the free-stream is sufficiently disturbed by vortical fluctuations, the laminar streaks quickly evolve into turbulence spots. The objective of the work is to obtain a mathematically rigorous description of the laminar streaks by means of perturbation methods. The work by [1] has shown that the spanwise velocity component of the free-stream vortical disturbances is responsible for the streak generation in the core of the boundary layer, while here we explain the mechanism of the penetration and confinement of the free-stream disturbances in the outer portion of the boundary layer. This analysis is important because (i) a more realistic streak profile may be obtained, (ii) the disturbances in the outer layer have been observed to trigger streak instability in recent numerical simulations [2], [3]. A further aim is to compare the results with the available experimental data by [4].

MATHEMATICAL FORMULATION

The dynamics of the laminar streaks is governed by the unsteady linearized boundary region equations, which are the rigorous asymptotic limit of the Navier-Stokes equation for low-frequency disturbances. The gist of the problem resides in the specification of the initial and outer boundary conditions, which properly account for the continuous mutual interaction between the boundary-layer and the free-stream disturbances.

RESULTS

The generation of the low-frequency laminar streaks by free-stream vortical disturbances may be explained through two different, independent physical mechanisms. In the first, discovered by [1], the *spanwise* velocity component of the free-stream gust is responsible for the amplification of the streamwise velocity perturbation in the core of the boundary layer. The gust streamwise velocity and the pressure disturbances do not play a role. Here we explain the second mechanism, which operates as follows. Through the Poisson equation in the edge-layer, the gust *streamwise* velocity induces a low-frequency *pressure* disturbance, which, in turn, drives a spanwise velocity fluctuation. The latter is finally balanced by a wall-normal velocity fluctuation. All these velocity components and the induced pressure fluctuation drive the outer portion of the laminar streaks inside the boundary layer.

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Figure 1 (left) shows the downstream evolution of the streamwise velocity profile of the laminar streaks. The peak inside the boundary layer is due to the first mechanism, while the velocity perturbation near the free-stream is caused by the second one. The result is a realist streak profile, similar to the ones often encountered in the experimental studies [5]. Figure 1 (right) presents the comparison between the experimental profiles of the fluctuating energy by [4] and our calculations, rescaled by the maxima of each experimental profile. The agreement is good and the low-frequency fluctuations penetrate more deeply into the boundary layer and are amplified more than the high-frequency fluctuations, which are confined in the outer portion of the boundary layer. Realistic assumptions based on the experimental campaign have been made, such as small streak amplitude and homogeneous free-stream turbulence along planes perpendicular to the mean flow direction.

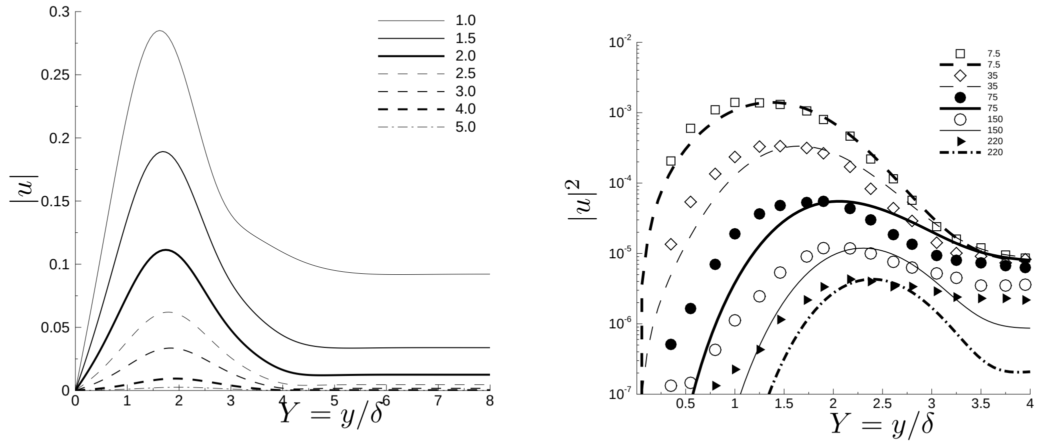

Figure 1. Amplitude of streamwise velocity profiles for $\{\kappa, \kappa_2\} = \sqrt{2\pi\nu\lambda_x^*/U_\infty}/\{\lambda_z^*, \lambda_y^*\} = \{1, 1\}$, $k_1 = 2\pi\lambda_z^*/\lambda_x^* = 0.1$, $\hat{u}_3^\infty = -0.2$ at $\overline{x} = 2\pi x^*/\lambda_x^* \geq 1$ (left), and comparison with experimental profiles of fluctuating energy for frequencies $\hat{F}=10^6\omega^*\nu/U_\infty^2$ (right). Lines denote the numerical calculations and symbols the experimental data at $x^* = 500$ mm for $U_\infty = 8$ m/s shown in figure 12a at page 210 of [4] (ν is the kinematic viscosity, $\lambda^*_{x,y,z}$ are the gust streamwise, wall-normal and spanwise wavelengths, U_∞ is the mean free-stream velocity, $\hat{u}_3^\infty$ is the spanwise velocity of the gust, ω^* is the frequency).

CONCLUSIONS AND OUTLOOK

The present research effort has been successful to gain further insight into the formation of the laminar streaks by free-stream vortical fluctuations. Further work should be directed at investigating if and how the boundary layer disturbances in the outer layer trigger streak instability and lead the streaks to the breakdown into turbulent spots.

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