Nonlinear development of Klebanoff modes in a laminar boundary layer

Pierre Ricco[1] and Xuesong Wu[2]

[1] Institute for Mathematical Sciences, Imperial College pierre.ricco@ic.ac.uk
[2] Department of Mathematics, Imperial College x.wu@ic.ac.uk

1 Description of the problem

Experiments [1],[2] indicate that free-stream turbulence may penetrate into the boundary layer, forming streamwise elongated streaks, also called "Klebanoff modes". The latter may breakdown causing bypass transition to turbulence. Streaks induced by sufficiently small-amplitude free-stream fluctuations, modelled by convective gusts, have been investigated by [3] using the unsteady linearized boundary *region* equations. In the present work, we consider free-stream turbulence of moderate level, for which the induced streaks show a nonlinear behaviour. The nonlinear calculation is important as it is a prerequisite for analysing the secondary instability of the streaks.

2 Mathematical formulation

The main parameters are (i) ϵ, the amplitude of the free-stream disturbances w.r.t. the mean flow velocity U_∞, and (ii) $R_\Lambda = U_\infty \Lambda/\nu$, where Λ is the spanwise length scale and ν is the kinematic viscosity. We assume *low frequency* (ω) (or, equivalently, long-wavelength) disturbances, which are the ones that primarily penetrate into the boundary layer. Viscous diffusion along the spanwise direction is relevant as the Klebanoff modes are studied at a downstream distance where $\delta = \mathcal{O}(\Lambda)$, where δ is the boundary layer thickness. The mathematical framework is thus the nonlinear unsteady boundary region equations (NLUBR), which are a rigorous asymptotic limit of the Navier-Stokes equations with the spanwise (z) viscous diffusion retained, but the streamwise (x) viscous diffusion neglected. These equations are parabolic in the streamwise direction and require appropriate initial and boundary conditions. The initial conditions are found by solving the $x \ll 1$ limit of the NLUBR. The outer (free-stream) boundary conditions are determined by asymptotic matching with the nonlinearly evolving free-stream vortical disturbances, which are not independent from their viscous counterpart as they are continuously affected by the boundary layer displacement. The free-stream fluctuations are modelled by two convective gusts with the same streamwise $k_1 = \omega\Lambda/U_\infty$ but opposite spanwise $k_3 = 2\pi/\Lambda$ wavenumbers $(k_1, k_3) = (1, \pm 1)$.

We consider $\epsilon R_\Lambda = \mathcal{O}(1)$, i.e. nonlinearity plays a key role. The present work can thus be seen as an extension of the linearized case [3] ($\epsilon R_\Lambda \ll 1$). A second-order finite-difference scheme, which is backward in x and central along the vertical direction η is used. The nonlinear terms are evaluated by the pseudo-spectral method with Fourier transformation in time and along z. A second-order predictor-corrector scheme is employed for their correct evaluation.

3 Results

Nonlinear interactions generate higher harmonics $(2, \pm 2), (3, \pm 3)$..., a mean flow distortion $(0, 0)$, and a spanwise periodic mean flow distortion $(0, \pm 2), (0, \pm 4)$..., which can be referred to as *steady* streaks. Nonlinearity is not influential during the first stage of the evolution, but eventually it has a stabilizing effect on the growth of the Klebanoff modes, as shown in figure 1 (left). The oscillatory components of higher harmonics are shown in figure 1 (right) for the streamwise velocity component. The mean flow distortion $(0,0)$ shows an higher velocity magnitude than the Blasius flow near the wall and lower in the upper portion of the boundary layer.

The instantaneous streamwise velocity profiles, shown in figure 2 (left), take both positive and negative values w.r.t. the mean flow near the wall, while only negative values close to the free-stream. Inflection points in the $\eta - z$ plane which are maxima of the spanwise vorticity are detected in proximity of the wall. They could be presursors of inviscid instability [4],[5]. A contour of the instantaneous streamwise velocity is shown in figure 2 (right), which indicates that low-speed fluid is driven upward toward the free-stream.

Further work is in progress on the secondary instability of the Klebanoff modes to shed light on the mechanism of bypass transition, which is still not properly understood.

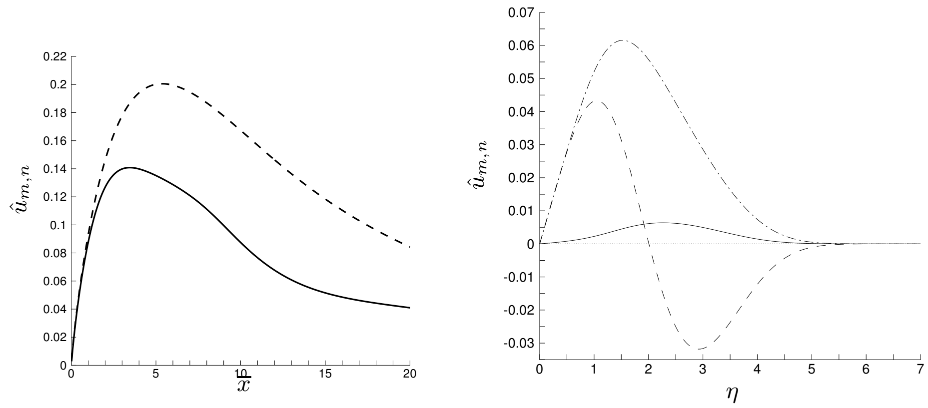

Fig. 1. Left: effect of nonlinearity on maximum r.m.s. of streamwise velocity ($k_1 = 0.05$, $R_\Lambda = 400$, $\epsilon = 0.01$). Solid lines: nonlinear case, dashed lines: linearized case. Right: profiles of the streamwise velocity of modes $(0,0)$ (dashed line), $(1,1)$ (dashed-dotted line) and $(2,2)$ (solid line) at $\overline{x} = k_1 x = 2$, $k_1 = 0.01$, $R_\Lambda = 400$ and $\epsilon = 0.005$.

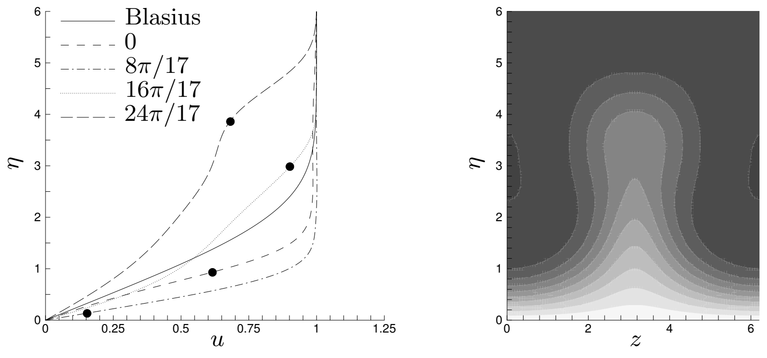

Fig. 2. Right: Instantaneous streamwise velocity profiles during different phases ϕ of the time modulation at $z = 0$ and $\overline{x} = 2$. $k_1 = 0.01, R_\Lambda = 400$, $\epsilon = 0.01$. Black dots indicate inflections points. Left: contour of instantaneous streamwise velocity in $\eta - z$ plane at $\overline{x} = 2$ ($\epsilon = 0.01$); darker colours indicate higher velocity.

References

1. J.H.M. Fransson, M. Matsubara, P. H. Alfredsson: J. Fluid Mech., **527** (2005).
2. M. Matsubara, P. H. Alfredsson: J. Fluid Mech., **430** (2001).
3. S. J. Leib, D. W. Wundrow, M. E. Goldstein: J. Fluid Mech., **426** (2001).
4. D. W. Wundrow, M. E. Goldstein: J. Fluid Mech., **380** (1999).
5. X. Wu, M. Choudhari: J. Fluid Mech., **483** (2003).