Imperial College London

Nonlinear development of Klebanoff modes in a laminar boundary layer

by Pierre Ricco[1] and Xuesong Wu[2]

pierre.ricco@imperial.ac.uk - http://www.ma.ic.ac.uk/~par - x.wu@imperial.ac.uk - http://www.ma.ic.ac.uk/~xwu

[1]Institute for Mathematical Sciences and [2]Department of Mathematics - Imperial College London

Objective

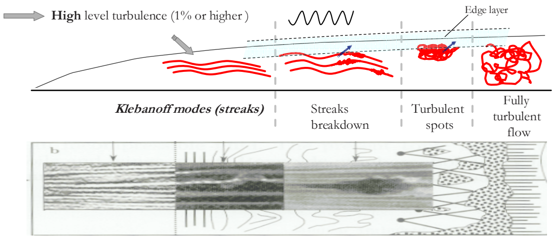
The purpose of our research is to study the influence of finite-amplitude free-stream turbulence on the transition to turbulence of laminar boundary layers.

Low frequency streaks (Klebanoff modes)

For relatively high free-stream fluctuations, low-frequency streamwise fluctuations penetrate into the boundary layer to form elongated streaks.

Instability of streaks and bypass transition

Streaks may eventually become unstable and develop into turbulent spots. The classical TS receptivity and instability mechanism is not relevant and we thus refer to it as bypass transition to turbulence by secondary instability.



Nonlinear boundary region (NBR) equations

The streaks dynamics is governed by the NBR eqs., which describe the origin of the streaks, their lift-up and the distortion of the Blasius boundary layer. The NBR eqs. are an extension of the boundary layer equations, in that the spanwise ellipticity of the fluctuations is accounted for because the thickness of the boundary layer is of the same order of the spanwise length scale.

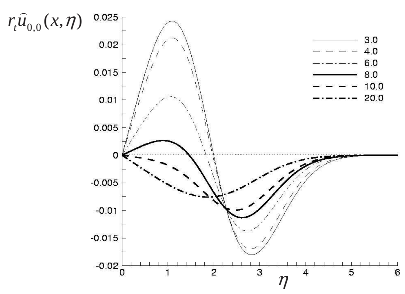
The outer boundary conditions (i.e. $y \to \infty$) are determined by asymptotic matching between the boundary layer disturbances and two free-stream vortical gusts with opposite spanwise wavenumbers. The displacement on the vortical gusts given by the streamwise growth of the boundary layer and the viscous dissipation of the gusts intensity are both taken into account.

Flow solution

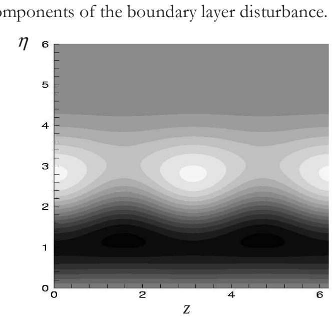
As the flow is periodic along the spanwise (z) direction and in time, the solution can be expressed in the form of a double Fourier series, as follows

$$\{u, v, w, p\} = \{U, V, 0, -1/2\} + r_t \sum_{m,n=-\infty}^{+\infty} \{\hat{u}_{m,n}(x,\eta), \hat{v}_{m,n}(x,\eta), \hat{w}_{m,n}(x,\eta), \hat{p}_{m,n}(x,\eta)\} e^{imk_1 t + ink_3 z}$$

Mean flow distortion

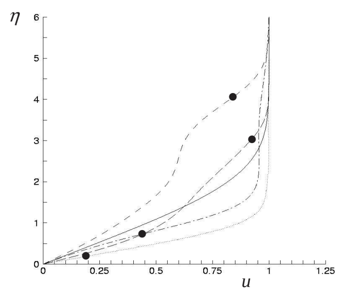


The streamwise mean-flow distortion $r_t\hat{u}_{0,0}(x,\eta)$ is positive near the wall and negative close to the free-stream during the beginning of the streamwise evolution. It eventually becomes negative throughout the boundary layer. The steady streaks show a much slower decay than the unsteady components of the boundary layer disturbance.

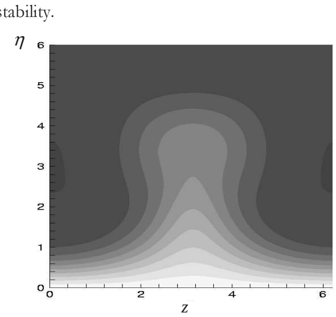


The contour of streamwise steady streaks at $x = 3$ shows that a spanwise modulation is generated by the nonlinear interactions. Light and dark colours indicate lower and higher velocity compared to the Blasius profile.

Instantaneous flow field



Both forward and backward fluctuations (compared to the Blasius profile, indicated by the solid line) occur near the wall, while in the outer part of the boundary layer only the latter appear. We further detect *inflection points* (black dots in graph) near the wall and the free-stream, which could be precursors of secondary instability.



The above contour of instantaneous streamwise velocity reveals that near-wall low-speed fluid moves toward the free-stream. At a different phase high-speed fluid is brought toward the wall from the free-stream.

Acknowledgements

Part of the research work was carried out by PR during the 2004 Summer Research Program of the Center for Turbulence Research at Stanford University. The collaboration with Professor Paul Durbin and Dr Tamer Zaki is greatly acknowledged.