Generation of highly-oblique T-S waves in a laminar boundary layer by free-stream turbulence

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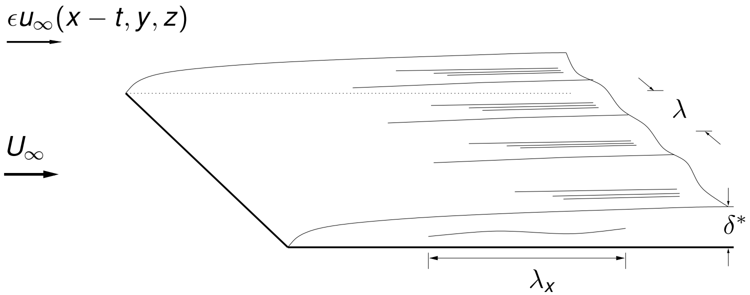
FLOW FIELD



- Compressible laminar boundary layer $M = U_\infty/c_\infty = \mathcal{O}(1)$
- Free-stream vorticity gust: $u_\infty = \hat{u}_\infty e^{ik_1x - i\omega t + ik_2y + ik_3z}$
- Low frequency
- Small amplitude $\epsilon \ll 1$
- No wall roughness

Definition

Excitation of instability waves in a laminar boundary layer by external agents

Examples:

- Leading-edge: Lam-Rott decaying eigensolutions turn into growing TS-waves (Goldstein 1983).
- FST-roughness: FST interacts with wall roughness to excite TS-waves (Goldstein 1985 - Wu 2001).
- FS vorticity-sound: FS vorticity interacts with FS sound to excite TS-waves (Wu 1999).

- Equations
  - Boundary region equations (Kemp 1951 - Leib *et al.* 1999)
  - Compressible
  - Unsteady
  - Linearized

- Outer boundary conditions
  - Asymptotic matching with free-stream vorticity gust

$$\frac{\partial w}{\partial \eta} + |\kappa|(2\overline{x})^{1/2} w \rightarrow i\kappa_2(2\overline{x})^{1/2} \mathrm{e}^{i\left(\overline{x}+\kappa_2(2\overline{x})^{1/2}\eta\right)} \mathrm{e}^{-\left(\kappa^2+\kappa_2^2\right)\overline{x}}$$

Key parameter

$\kappa \approx \delta^*/\lambda$

- δ^* Boundary-layer thickness
- λ Gust spanwise wavelength

- $\kappa = 0$ Disturbances grow initially and persist indefinitely
- $\kappa = \mathcal{O}(1)$ Disturbances grow and decay by viscosity $\sim \kappa^2$
- $\kappa \ll 1$ TS WAVES - Triple-deck interactive regime

FLOW RESPONSE REGIMES

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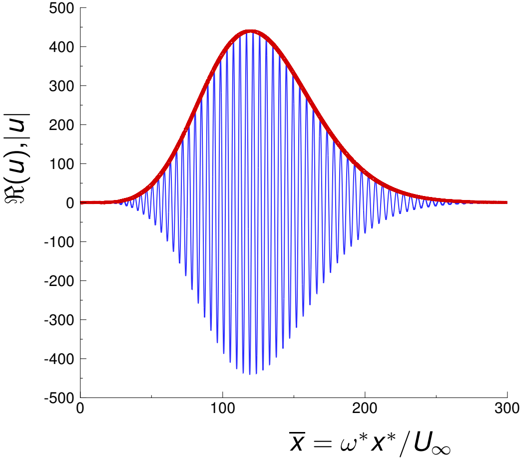
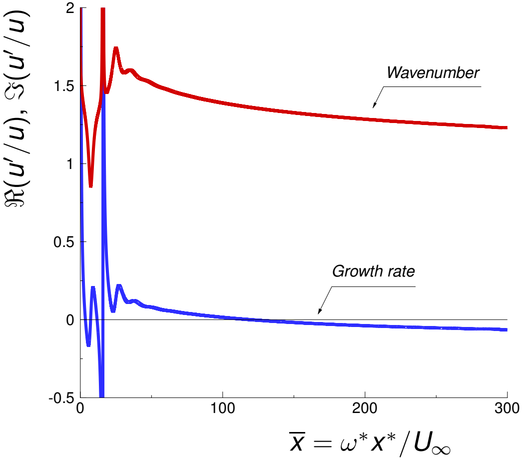
TS-WAVES EXCITATION: NUMERICAL EVIDENCE



- $\kappa = 0.02$
- $M = 3$
- $\overline{x} = \mathcal{O}(\kappa^{-1})$

GROWTH RATE and WAVENUMBER



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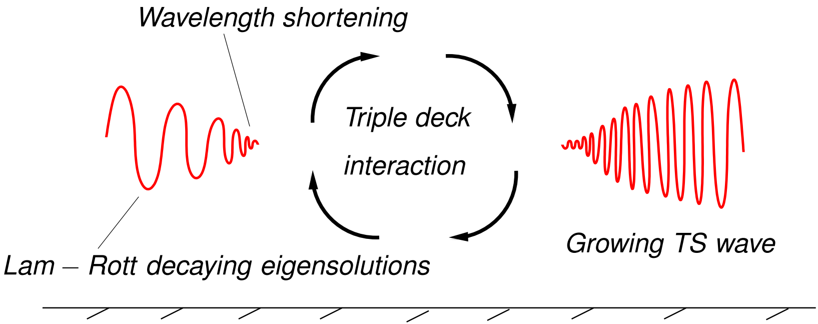
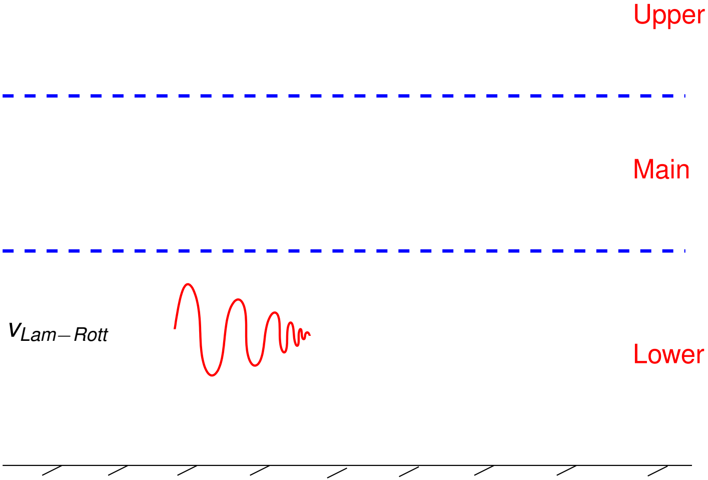
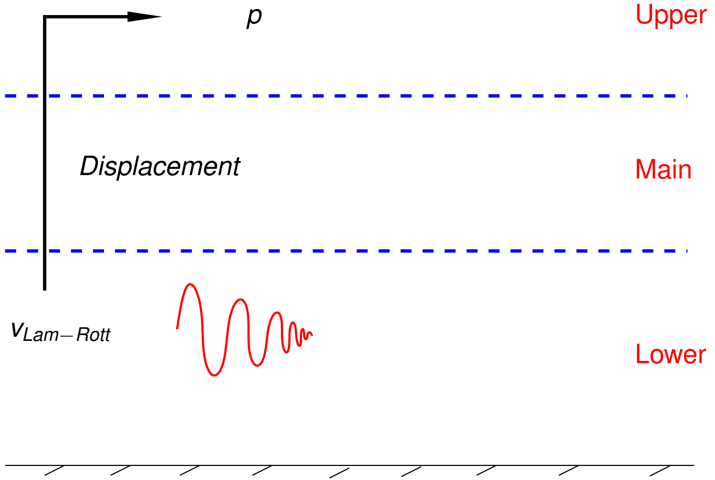
RECEPTIVITY: WAVELENGTH SHORTENING - I



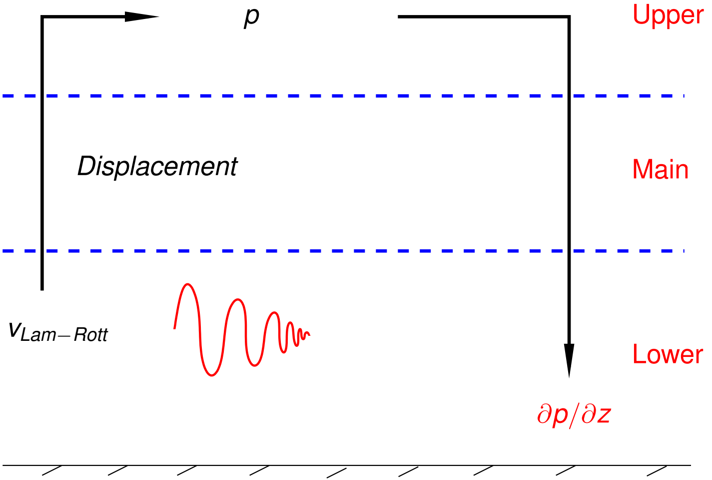
$$\overline{x} = \omega^* x^* / U_\infty = \mathcal{O}(\kappa^{-1})$$


Upper
Main
$v_{Lam-Rott}$
Lower

RECEPTIVITY: WAVELENGTH SHORTENING - II



RECEPTIVITY: WAVELENGTH SHORTENING - II



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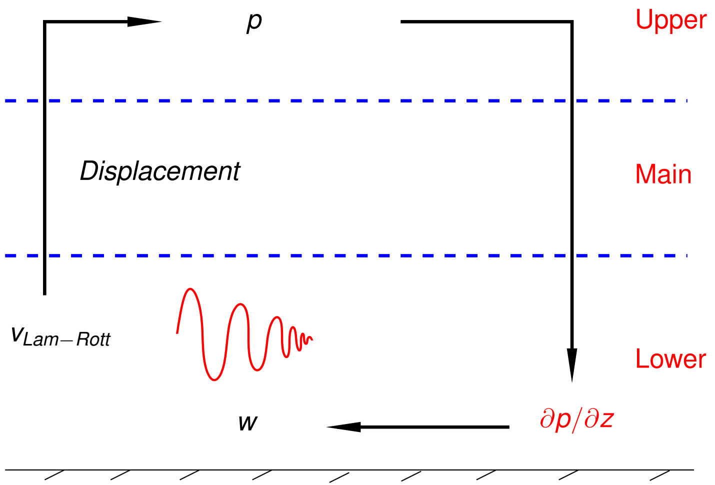
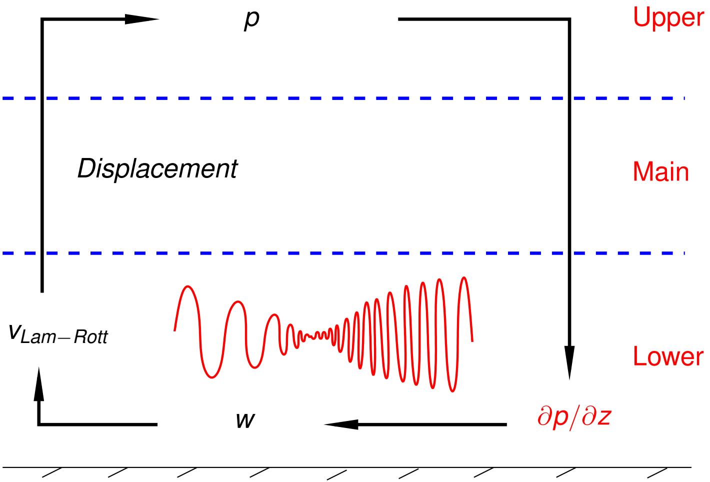



p
Upper
Displacement
Main
$v_{Lam-Rott}$
Lower
w
$\partial p/\partial z$

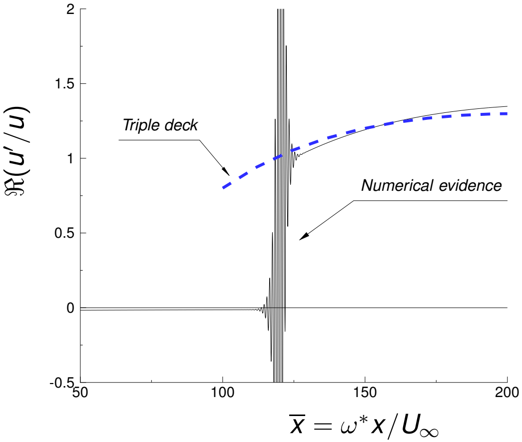
TRIPLE-DECK EIGENRELATION

$$u \sim \exp\left(\frac{i}{\kappa^b}\int_0^{\overline{x}} \alpha_1(x_1)\mathrm{d}\check{x}\right)$$

$$\Delta(x_1,\alpha_1) \equiv \int_{\eta_0}^{\infty} \mathrm{Ai}(\check{\eta})\mathrm{d}\check{\eta} - \frac{\mu_w^{1/3}}{T_w^{7/3}}\left(\frac{F''(0)}{\sqrt{2x_1}}\right)^{5/3}(i\alpha_1)^{-1/3}\,\mathrm{Ai}'(\eta_0) = 0$$

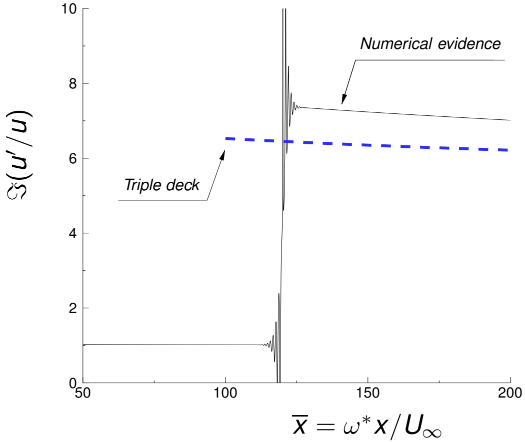
- $\Re(\alpha_1)$ wavenumber
- $\Im(\alpha_1)$ growth rate
- x_1 scaled streamwise coordinate
- η, η_0 scaled wall-normal coordinates
- Ai complex Airy function (Gil *et al.* 2001)
- Compressibility enters through μ_w, T_w and $F''(0)$

THEORY vs. NUMERICAL EVIDENCE - I



- $\kappa = 0.0005$
- $M = 3$

THEORY vs. NUMERICAL EVIDENCE - II



- $\kappa = 0.0005$
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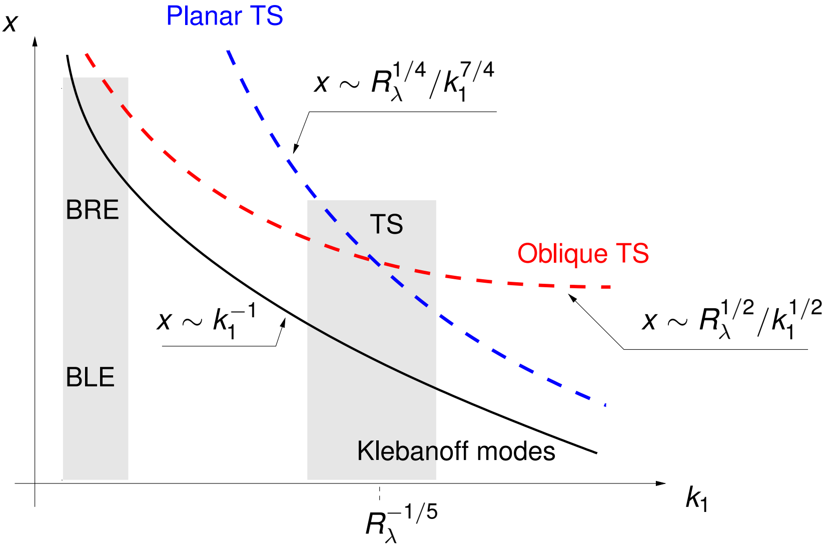
KLEBANOFF MODES and TS WAVES



SUMMARY

- New leading-edge receptivity mechanism
- Lam-Rott eigensolutions involved
- Wavelength shortening

Key difference

$\partial p/\partial z$ is active
$\partial p/\partial x$ is Goldstein 1983's mechanism

- Pratically important: $M = 3$, $\lambda = 0.02$ m $\rightarrow$ $x = 0.15$ m