REDUCTION OF TURBULENT FRICTION BY HYDROPHOBIC SURFACES WITH SHEAR-DEPENDENT SLIP LENGTHS

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INTRODUCTION

There has recently been a surge in the study of turbulent drag reduction techniques based on surfaces called superhydrophobic (SH), obtained by combining geometrical and chemical interface properties. Small-scale systems design makes such surfaces promising candidates for various industrial applications involving aqueous flows. The reduction of wall friction is caused by the wall slip between the turbulent flow of the fluid and the solid surface.

The direct numerical simulations (DNS) in [7] have shown a maximum drag reduction of 26%. The wall boundary condition (BC) was enforced by the use of a constant slip length, as follows: $u_{wall} = l_s u_{\mathrm{y,wall}}$, where $u_w all$ is the slip wall velocity, l_s is the constant slip length, and the subscript y indicates partial derivative with respect to the wall-normal direction.

However, experiments by [2] suggest that SH surfaces may be characterized by slip length which are *wall-shear dependent*. For the surfaces studied by [2] and [3], slip lengths of the order of one hundred of microns have been measured. The large slip length may be partly due to the fact that the slip length *increases* with the wall-shear stress. This is very interesting for marine applications because, at such high Reynolds numbers, the wall-shear stress is severe.

Even though further experimental verification for the shear-dependent SH surfaces is urgently needed, we thought that it would be interesting to model these surfaces numerically and to investigate their turbulent drag reduction property. Ours is the first DNS of wall turbulence modified by these shear-dependent surfaces. The Reynolds is $Re_\tau = u^*_\tau h^*/\nu^* = 180$, where u^*_τ is the friction velocity in the uncontrolled case, h^* is half the channel height and ν^* is the kinematic viscosity.

Two theoretical approaches have been used to guide our numerical results and predict the value of turbulent drag reduction computed through DNS. In the first approach, Lyapunov stability analysis is used to extract the feedback control law of hydrophobic surface with the shear-dependent slip length. This is the first time that the control law found through this method is i) extended to the three dimensional case and ii) conceptually linked with the properties of hydrophobic surfaces. In the second approach, the turbulent drag reduction given by the hydrophobic surfaces is estimated through an extension of the methodology first proposed by [5]. The estimates match the DNS results very well. The theoretical frameworks are first introduced and the numerical results are then presented.

THEORETICAL RESULTS

Control by Lyapunov stability analysis

The objective of this theoretical part is to stabilise a turbulent channel flow about the corresponding Poiseuille flow at the same mass flow rate. The analysis, carried out in 3D for the first time, is based on the 2D work by [1].

The mathematical formulation for the BCs is linked to SH surfaces for the first time and is extended to the shear-dependent slip length model. The disturbance energy of the system is expressed through the L^2 norm of the velocity vector field: $E(\mathbf{w}) = \|\mathbf{w}\|_{L^2}$. For the 3D case, the stability analysis readily relates to the three cases considered in [7]. In the constant slip length case [1], the streamwise-velocity BC is $u(x, \mp 1, z) = \pm k u_y(x, \mp 1, z)$ (analogous for the spanwise velocity w). We impose $k = a u_y + b$, a prove global stability in L^2 norm as the time derivative of $E(\mathbf{w})$ is computed and upper-bounded:

$$\dot{E}(\mathbf{w}) \leq -\frac{\alpha E(\mathbf{w})}{2} - \frac{2}{Re_p}\left[\frac{2}{b\left(1+\sqrt{1+\frac{4a}{b^2}}\right)} - 1\right] \times \int_0^{L_z}\int_0^{L_x} \left[u^2(x,-1,z) + w^2(x,-1,z)\right] \mathrm{d}x\,\mathrm{d}z - \frac{4}{bRe_p\left(1+\sqrt{1+\frac{4a}{b^2}}\right)} \int_0^{L_z}\int_0^{L_x} \left[u^2(x,1,z) + w^2(x,1,z)\right] \mathrm{d}x\,\mathrm{d}z, \quad (1)$$

where $\alpha = 1/Re_p - 4$. For $\alpha > 0$, i.e. $Re_p < 1/4$, $E(\mathbf{w})$ decays exponentially in time in the uncontrolled case. As discussed in [1], despite this limitation the control is effective at the (much higher) Re_p used in the DNS. Our analysis shows that stability enhancement leads to the condition $a \leq 1 - b$.

The Fukagata-Kasagi-Koumoutsakos method

The derivation for is carried out following the method of [5] is extended to the shear-dependent slip length case, leading to the following:

$$a_x(1-\mathcal{R})\frac{Re^3_{\tau 0}}{Re_p} + b_x Re_{\tau 0} = \left(\frac{1}{\kappa}\ln Re_{\tau 0} + F\right)\left(\frac{1-\sqrt{1-\mathcal{R}}}{1-\mathcal{R}}\right) - \frac{\kappa \ln\sqrt{1-\mathcal{R}}}{\sqrt{1-\mathcal{R}}}, \quad (2)$$

where $\mathcal{R} = 1 - (u^*_\tau/u^*_{\tau 0})^2$ is the drag reduction, Re_p is the Reynolds number based on the Poiseuille velocity flow and h^*, and κ is the von Kármán constant.

NUMERICAL RESULTS

Eq. (2) is solved numerically and, as shown in figure 1, the data match the DNS results very well. Fig.1 shows that for a fixed b_x value, as $a_x \to 0$, the drag reduction values found with the the constant slip length [7] are recovered. It

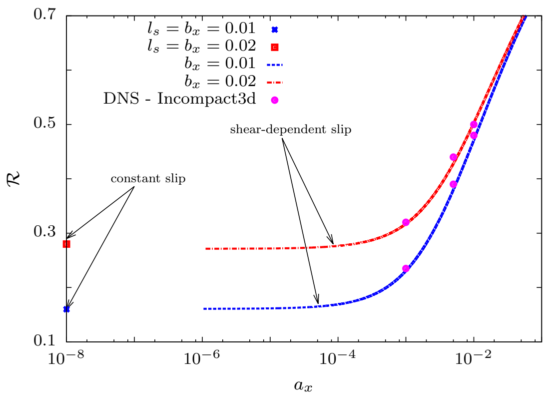

Figure 1: Theoretical prediction from Eq.(2) compared with DNS computation.

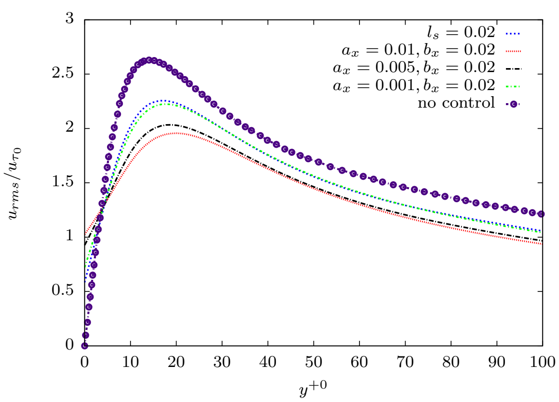

Figure 2: Streamwise velocity r.m.s. for the three models.

is therefore proven that a shear-dependent slip length can significantly enhance the drag reduction. Figure 2 shows that the fluctuations are much more decreased as a_x increases for a fixed b_x, implying an increase in the effective slip length in the the shear-dependent case.

The PDF for the streamwise fluctuating velocity in the no-slip, constant and shear-dependent slip length cases are computed (not shown). The excursions of u are reduced the most in the shear-dependent case. The reduction of high-speed u fluctuations cause a reduction of ω_z fluctuations because this quantity is dominated by u_y. Therefore, if such fluctuations are damped, the wall-shear stress drag is also be reduced.

The control effect is also analysed through the orientation of the vorticity vector $\boldsymbol{\omega}$ and the principal strain rates denoted by s_i, $i \in [\![1,3]\!]$. The associated eigenvectors are the principal axes denoted by $\boldsymbol{e}_i$. The angles between $\boldsymbol{\omega}$ and $\boldsymbol{e}_i$ are defined by $\cos\theta_i = \boldsymbol{\omega}\cdot\boldsymbol{e}_i/(\|\boldsymbol{\omega}\|\|\boldsymbol{e}_i\|)$ subsequently related to the enstrophy production term, $\omega_i S_{ij}\omega_j = \omega^2 s_i \cos^2\theta_i$ [6]. In a control framework, the evolution of enstrophy is often used to highlight its effect. Near the wall, the finite slip length models damp the enstrophy production term.

The impact of control is further quantified by computing the weighted contributions of the Reynolds shear-stress arising in the FIK identity [4] for each quadrant. In Fig.4, Q2 and Q4 events are reduced the most in the shear-dependent case.

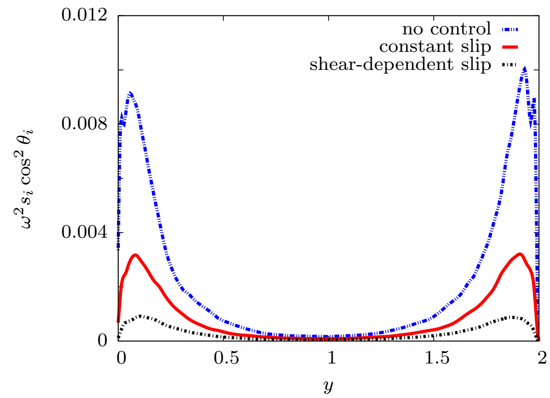

Figure 3: Enstrophy production term for the three models.

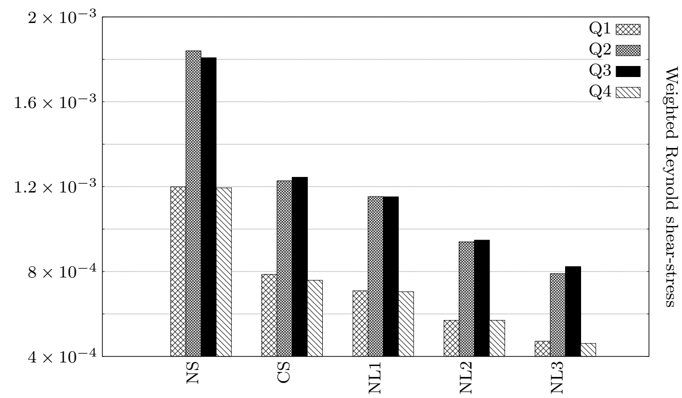

Figure 4: Weighted Reynolds shear-stress for no-slip case (NS), constant slip (CS) with $l_s = 0.02$ and shear-dependent slip (NL1) using $a_x = 0.001, b_x = 0.02$, (NL2) with $a_x = 0.005, b_x = 0.02$ and (NL3) with $a_x = 0.01, b_x = 0.02$.

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