OPTIMAL CONTROL OF STREAKS INDUCED BY FREE-STREAM TURBULENCE IN INCOMPRESSIBLE BOUNDARY LAYER BY WALL BLOWING AND SUCTION

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Abstract A framework for active control of spatially developing flows is developed and it is applied to suppress the energy growth of streaks developed within an incompressible boundary layer due to free-stream turbulence. The developed control framework uses the primitive variables, velocity and pressure. The flow model is based on the linearised unsteady boundary-region (LUBR) equations, which is the rigorous asymptotic limit of the Navier-Stokes equations. The effect of free-stream-turbulence at a particular wavenumber appears as explicit forcing of these equations and is obtained by asymptotic matching with the far field conditions. Optimal control theory is used to minimise a cost function based on the energy of streaks and the actuation is by blowing and suction at the wall. A special procedure is developed within the control framework to account for the explicit external forcing. It is shown that it is possible to reduce the peak streak energy by $5-6$ times.

INTRODUCTION

Active flow control has emerged as a major research area in the past decade. In the present work, we use as flow model the LUBR equations that have been shown to represent well the initial (linear) stages of streak growth in a boundary layer [1]. Turbulence is represented as a superposition of vortical free-stream Fourier modes with wavenumbers k_1, k_2 and k_3 in the streamwise, transverse and spanwise directions, respectively. A controller is designed for this flow system using optimal control theory and a quadratic cost funtion. This approach had been used successfully to control linearly unstable Tollmien-Schlichting waves and to reduce the transient energy growth of streamwise vortices in channel flow [2].

METHODOLOGY

We consider the flow of uniform velocity U_∞ over an infinitely-thin flat plate due to an homogeneous, statistically-stationary turbulence field, as shown in figure 1. The flow can be divided into four asymptotic regions as explained by [1] and shown also in figure 1. In the present work we are interested in region III, where the size of the boundary layer thickness is of the same order as the spanwise length scale, Λ. In this region, the streak growth is governed by the linearised Navier-Stokes equations (about the Blasius profile) that retain the pressure and viscous terms in the wall-normal and spanwise directions. These equations are called linearised unsteady boundary-region (LUBR) equations [1]. The boundary conditions at the top of the boundary layer are obtained by asymptotic matching of the LUBR equations with the far field conditions, and take the form:

$$\bar{u} \to 0; \qquad \frac{\partial \bar{v}}{\partial \eta} + |\kappa| (2\bar{x})^{1/2} \bar{v} \to -e^{i(\bar{x}+\kappa_2(2\bar{x})^{1/2}\bar{\eta})} e^{-(\kappa^2+\kappa_2^2)\bar{x}} \tag{1}$$

$$\frac{\partial \bar{w}}{\partial \eta} + |\kappa| (2\bar{x})^{1/2} \bar{w} \to i\kappa_2 (2\bar{x})^{1/2} e^{i(\bar{x}+\kappa_2(2\bar{x})^{1/2}\bar{\eta})} e^{-(\kappa^2+\kappa_2^2)\bar{x}}; \qquad \frac{\partial \bar{p}}{\partial \eta} + |\kappa| (2\bar{x})^{1/2} \bar{p} \to 0 \tag{2}$$

as $\eta \to \infty$, where $\bar{x} = k_1 x$ (with $x = x^*/\Lambda$) is the scaled streamwise distance and $\kappa = k_3/(k_1 R_\Lambda)^{1/2}$, $\kappa_2 = k_2/(k_1 R_\Lambda)^{1/2}$ (with $R_\Lambda = U_\infty \Lambda/\nu$) are the scaled spanwise and transverse wavenumbers respectively. The resulting linear system has the form $\boldsymbol{E}\frac{\partial \boldsymbol{q}}{\partial \bar{x}} = \boldsymbol{A}(\bar{x})\boldsymbol{q} + \boldsymbol{f}(\bar{\boldsymbol{x}})$ where $\boldsymbol{q} = [\bar{u}\ \bar{v}\ \bar{w}\ \bar{p}]^T$ and $\boldsymbol{f}(\bar{\boldsymbol{x}})$ is a forcing vector due to free-stream-turbulence that has closed form analytic expression (equations $1, 2$). We augment appropriately the state vector $\boldsymbol{q}$ in order to bring the system to the more familiar form $\hat{\boldsymbol{E}}\frac{\partial \hat{\boldsymbol{q}}}{\partial \bar{x}} = \hat{\boldsymbol{A}}(\bar{x})\hat{\boldsymbol{q}}$. The system is discretised and the control theory for discrete systems is applied [3]. In the wall-normal direction we use rational Chebyshev polynomials and in the streamwise direction finite differences. The controllers are designed using optimal control theory to minimise a quadratic cost function, equal to the flow energy in the whole domain plus a control cost.

RESULTS

The methodology and code developed for the present work are validated against the reference results of [1] and very good matching is observed for the open loop case (no control). All the results presented below are for $\kappa = 1$ and $\kappa_2 = -1$. The profile of the amplitude of the wall-normal actuation velocity is shown in figure 1. The wall velocity starts to increase from the value of 0 at the tip of the plate ($\bar{x} = 0$), reaches a maximum at around $\bar{x} = 0.12$ and then smoothly approaches 0 further downstream.

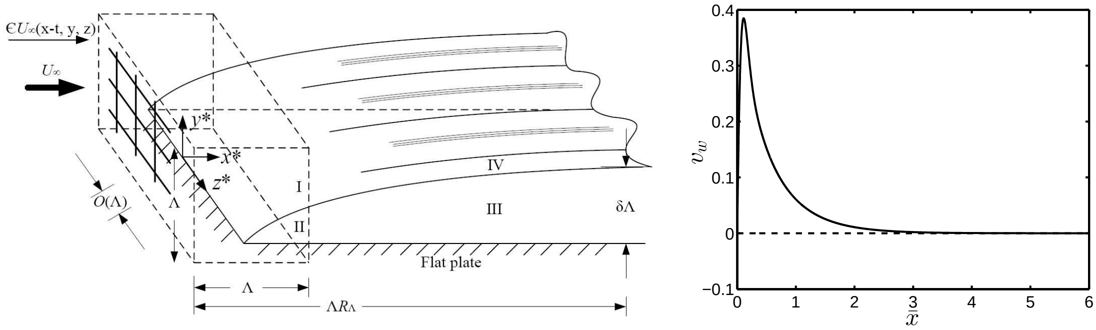

Figure 1. Flow configuration (*left*) and profile of the amplitude of the optimal wall-normal control velocity (*right*).

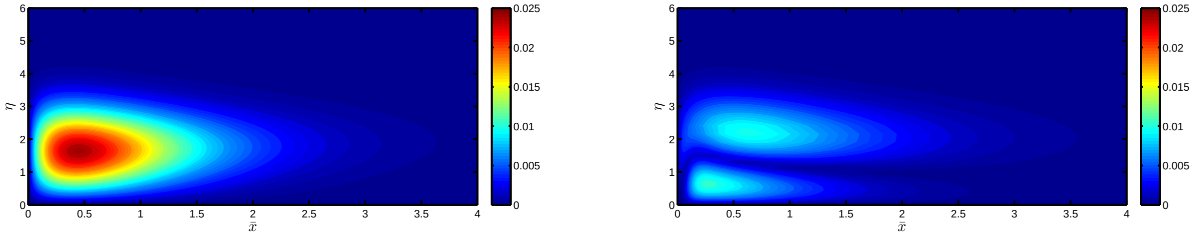

Figure 2. Contour plots of the amplitude of the streamwise perturbation velocity $\bar{u}$ without (*left*) and with control (*right*).

Figure 2 presents contour plots of the amplitude of the streamwise perturbation velocity $\bar{u}$ in the whole domain with and without control (same scales are used to facilitate comparison). The effect of control is to reduce the maximum value of the amplitude to 40% of the uncontrolled case. It is interesting to note that the effect of wall actuation (blowing and suction) is to lift the streak away from the wall and create a buffer vortex between the streak and the wall.

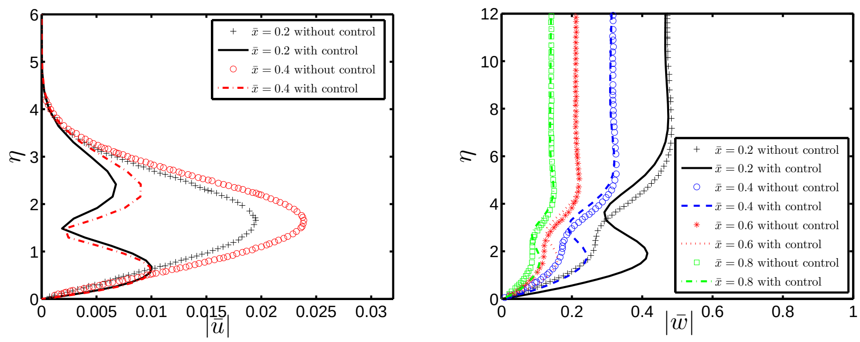

Figure 3. Amplitudes of the streamwise (*left*) and spanwise (*right*) perturbation velocities at various values of $\bar{x}$ with and without control.

Figure 3 shows the amplitudes of the streamwise and spanwise perturbation velocities with and without the control action for various streamwise locations. Note that the profile of $\bar{u}$ has two peaks in the controlled flow due to the presence of the buffer vortex shown earlier. Control affects the spanwise velocity only close to the wall, while far away the effect of wall action is diminished and the values are determined by the boundary conditions (i.e. equations 1 and 2), as expected. Note also that a peak appears in the spanwise velocity, located at a wall-normal distance between the two vortices.

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