Imperial College London

etc14

Optimal control of streaks induced by free-stream turbulence in incompressible boundary layer by wall blowing and suction: application for a linear streak model

Liang Lu[1], Pierre Ricco[2] & George Papadakis[1*]

[1]Department of Aeronautics, Imperial College London, London SW7 2AZ, UK

[2]Department of Mechanical Engineering, University of Sheffield, Sheffield S1 3JD, UK

* Email address for corresponding author: g.papadakis@imperil.ac.uk

Introduction

A framework for active control of spatially developing flows is developed and it is applied to suppress the energy growth of streaks developed within an incompressible boundary layer due to free-stream turbulence. The developed control framework uses the primitive variables, velocity and pressure. The flow model is based on the linearised unsteady boundary-region (LUBR) equations. The effect of free-stream-turbulence at a particular wavenumber appears as explicit forcing of these equations and is obtained by asymptotic matching with the far field conditions. Optimal control theory is used to minimise a cost function based on the energy of streaks and the actuation is by blowing and suction at the wall.

Mathematical formulation

We consider the flow of uniform velocity U_∞ over an infinitely-thin flat plate due to an homogeneous, statistically-stationary turbulence field. The flow can be divided into four asymptotic regions as explained by [1] and shown also in figure 1.

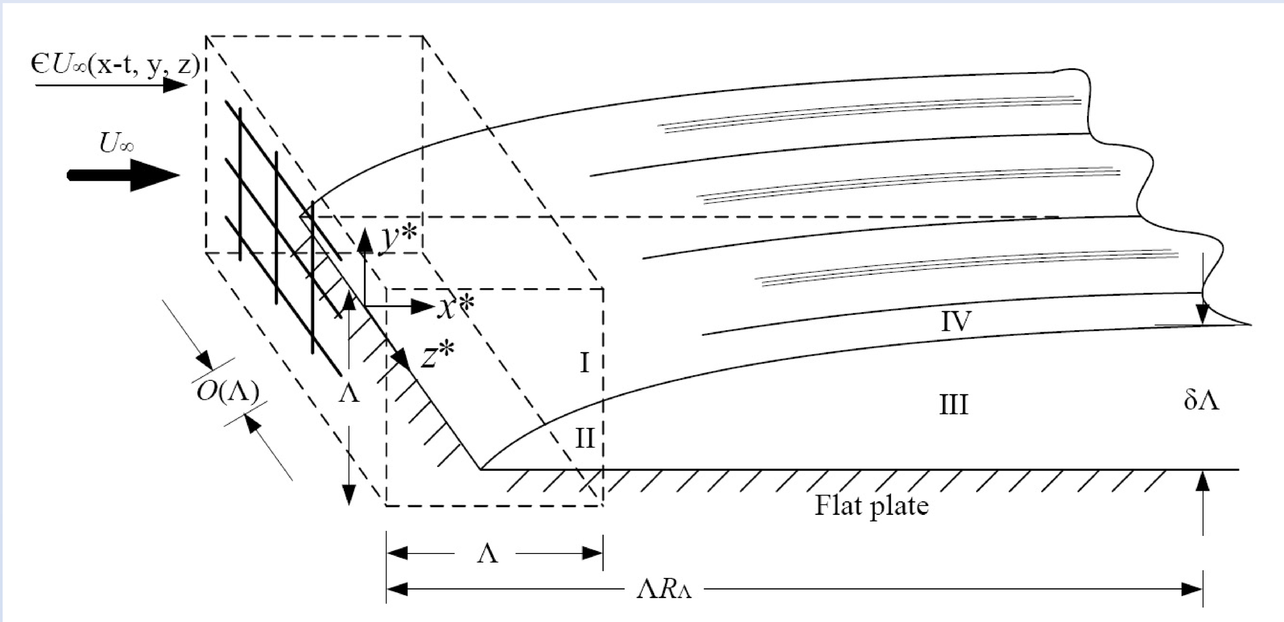

Figure 1: Flow configuration illustrating the asymptotic structure.

In the present work we are interested in region III, where the size of the boundary layer thickness is of the same order as the spanwise length scale, Λ. In this region, the streak growth is governed by the linearised Navier-Stokes equations (about the Blasius profile) that retain the pressure and viscous terms in the wall-normal and spanwise directions. These equations are called linearised unsteady boundary-region (LUBR) equations [1]:

$$-i\bar u + F'\frac{\partial \bar u}{\partial \bar x} - \frac{F}{2\bar x}\frac{\partial \bar u}{\partial \eta} - \frac{\eta F''}{2\bar x}\bar u + F''\bar v - \frac{1}{2\bar x}\frac{\partial^2 \bar u}{\partial \eta^2} + \kappa^2 \bar u = 0,$$

$$-i\bar v + F'\frac{\partial \bar v}{\partial \bar x} - \frac{F}{2\bar x}\frac{\partial \bar v}{\partial\eta} - \frac{1}{(2\bar x)^2}[\eta(\eta F')' - F]\bar u + \frac{(\eta F')'}{2\bar x}\bar v = -\frac{1}{2\bar x}\frac{\partial \bar p}{\partial \eta} + \frac{1}{2\bar x}\frac{\partial^2\bar v}{\partial\eta^2} - \kappa^2\bar v,$$

$$-i\bar w + F'\frac{\partial\bar w}{\partial\bar x} - \frac{F}{2\bar x}\frac{\partial \bar w}{\partial\eta} = \kappa^2\bar p + \frac{1}{2\bar x}\frac{\partial^2\bar w}{\partial\eta^2} - \kappa^2\bar w = 0,$$

$$\frac{\partial\bar u}{\partial\bar x} - \frac{\eta}{2\bar x}\frac{\partial\bar u}{\partial\eta} + \frac{\partial\bar v}{\partial\eta} + \bar w = 0,$$

The boundary conditions at the top of the boundary layer are obtained by asymptotic matching of the LUBR equations with the far field conditions, and take the form:

$$\bar u \to 0;\quad \frac{\partial\bar v}{\partial\eta} + |\kappa|(2\bar x)^{1/2}\bar v \to -e^{i(\bar x + \kappa_2(2\bar x)^{1/2}\eta)} e^{-(\kappa^2+\kappa_2^2)\bar x};$$

$$\frac{\partial\bar w}{\partial\eta} + |\kappa|(2\bar x)^{1/2}\bar w \to i\kappa_2(2\bar x)^{1/2} e^{i(\bar x+\kappa_2(2\bar x)^{1/2}\eta)} e^{-(\kappa^2+\kappa_2^2)\bar x};\quad \frac{\partial\bar p}{\partial\eta} + |\kappa|(2\bar x)^{1/2}\bar p \to 0,$$

as $\eta \to \infty$, where $\bar x = k_1 x$ (with $x = x^*/\Lambda$) is the scaled streamwise distance and $\kappa = k_3/(k_1 R_\Lambda)^{1/2}$, $\kappa_2 = k_3/(k_1 R_\Lambda)^{1/2}$ (with $R_\Lambda = U_\infty \Lambda/\nu$) are the scaled spanwise and transverse wavenumbers respectively. The resulting linear system has the form $\boldsymbol{E}\frac{\partial \boldsymbol{q}}{\partial \bar x} = \boldsymbol{L}(\bar x)\boldsymbol{q} + \boldsymbol{f}(\bar x)$ where $\boldsymbol{q} = [\bar u\ \bar v\ \bar w\ \bar p]^T$ and $\boldsymbol{f}(\bar x)$ is a forcing vector due to free-stream turbulence. For discretisation, we use rational Chebyshev polynomials in the wall-normal direction and finite differences in the streamwise direction.

Control synthesis and open-loop results

After discretisation, the system can be written as

$$\boldsymbol{q}_{i+1} = A_i \boldsymbol{q}_i + B_i \boldsymbol{u}_i + C_i,$$

where $\boldsymbol{u}_i = \frac{\partial \bar v_w(\bar x)}{\partial \bar x}$ which is the streamwise derivative of the wall-normal velocity at the wall is the control variable in the present study and C_i is the external disturbance matrix. Controllers are designed using optimal control theory to minimise a quadratic cost function, equal to the flow energy in the whole domain plus a control cost.

$$J_i = \frac{1}{2}\boldsymbol{q}_N^T P_N \boldsymbol{q}_N + \frac{1}{2}\sum_{i=0}^{i=N}\left(\boldsymbol{q}_i^T Q_i \boldsymbol{q}_i + \boldsymbol{u}_i^T R_i \boldsymbol{u}_i\right)$$

The control signal consists of two components, a feed-back part (that depends on the state vector) and a feed-forward part (that depends on the external forcing).

$$\boldsymbol{u}_i = -K_i \boldsymbol{q}_i + K_i^v (V_{i+1} - P_{i+1} C_i)$$

where K_i is feed-back gain and K_i^v is feed-forward gain. Open-loop results are presented in figure 2. Both the streamwise and spanwise velocity components at all the streamwise positions have good match with the results of Leib et al. (1999).

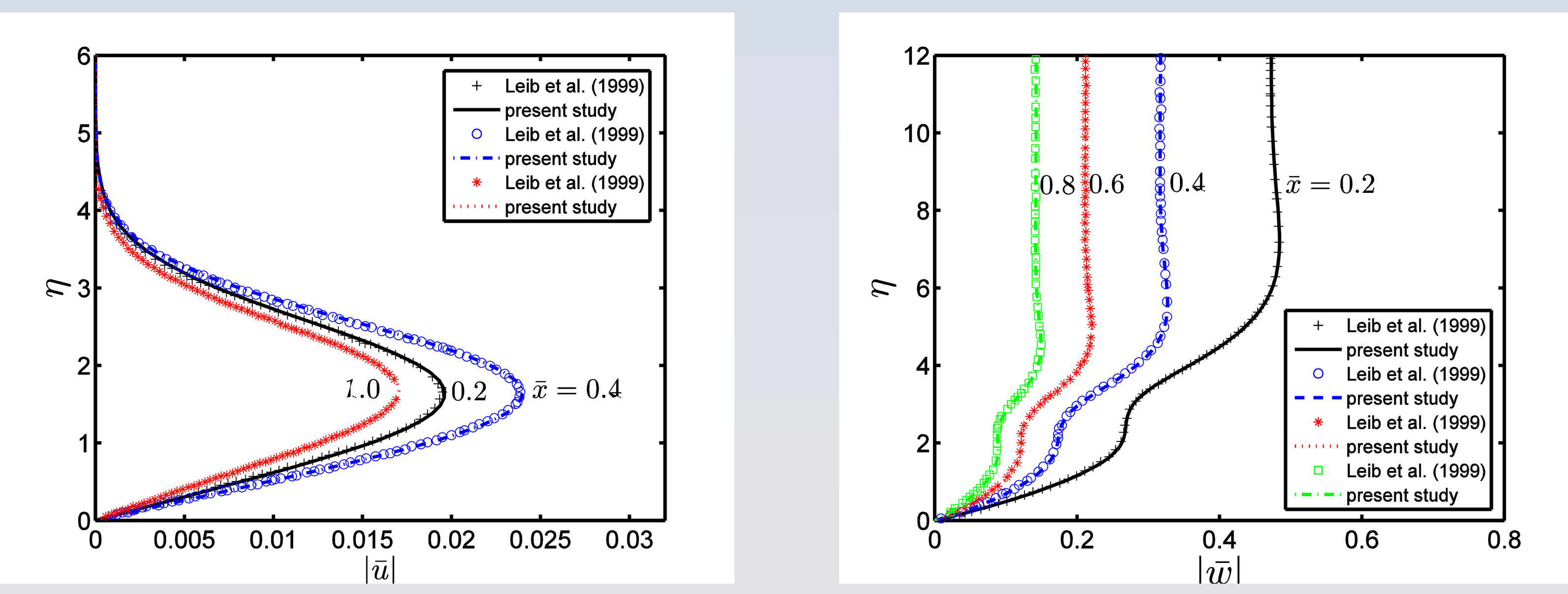

Figure 2: Profiles of the magnitudes of the (left) streamwise and (right) spanwise perturbation velocity at various values of $\bar x$.

Closed-loop results

All the results presented below are for $\kappa = 1$ and $\kappa_2 = -1$. The profiles of the wall-normal actuation velocity are shown in figure 3.

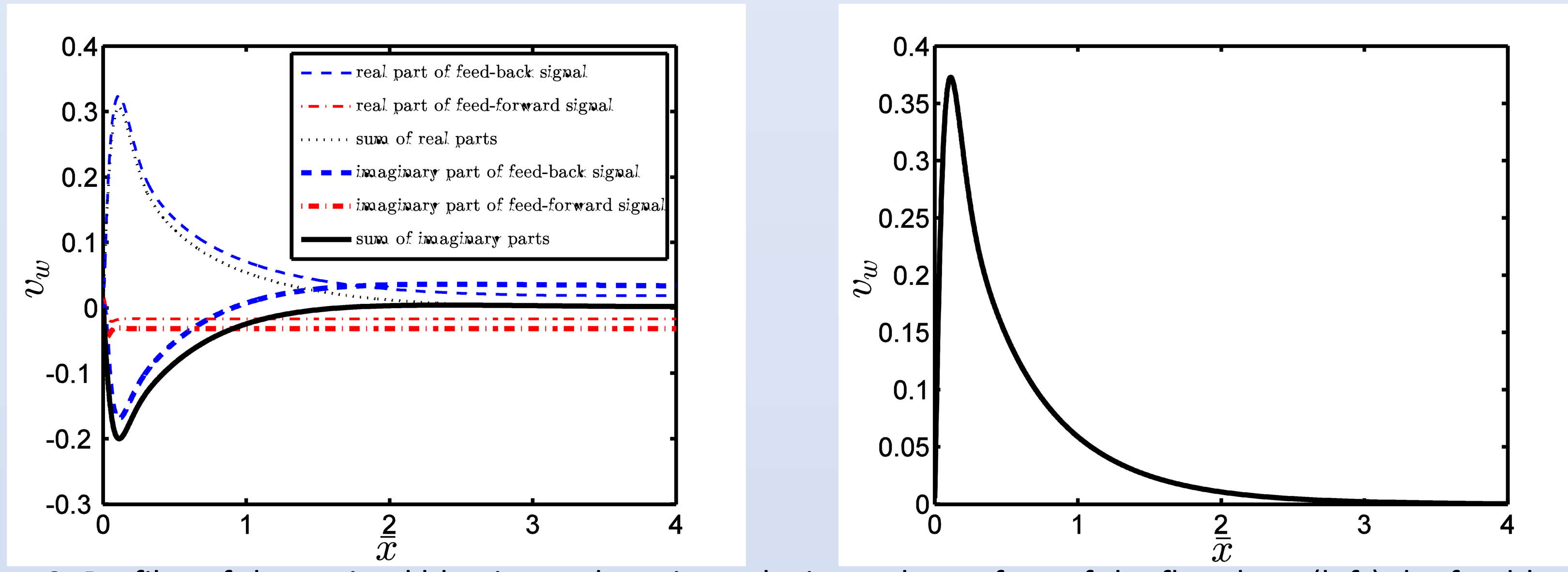

Figure 3: Profiles of the optimal blowing and suction velocity at the surface of the flat plate: (left) the feed-back and feed-forward parts and (right) the magnitude.

Figure 4 shows perturbation velocities with and without the control. The profile of $\bar u$ has two peaks in the controlled flow due to the presence of the buffer vortex. Control affects the spanwise velocity only close to the wall, while far away the effect of wall action is diminished and the values are determined by the boundary conditions. A peak appears in the spanwise velocity, located at a wall-normal distance between the two vortices.

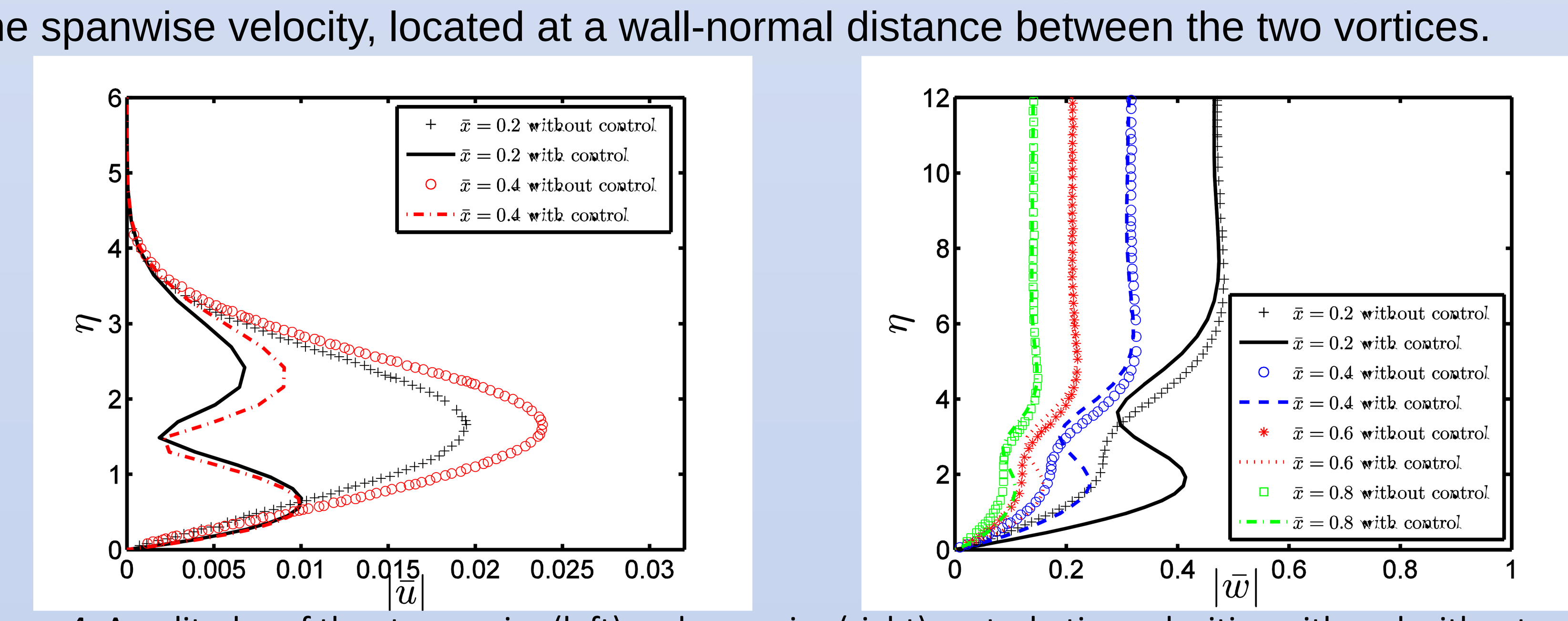

Figure 4: Amplitudes of the streamwise (left) and spanwise (right) perturbation velocities with and without control.

Figure 5 presents contour plots of the amplitude of the streamwise perturbation velocity in the whole domain with and without control. The effect of control is to reduce the maximum value of the amplitude to 40% of the uncontrolled case. It is interesting to note that the effect of wall actuation (blowing and suction) is to lift the streak away from the wall and create a buffer region between the streak and the wall.

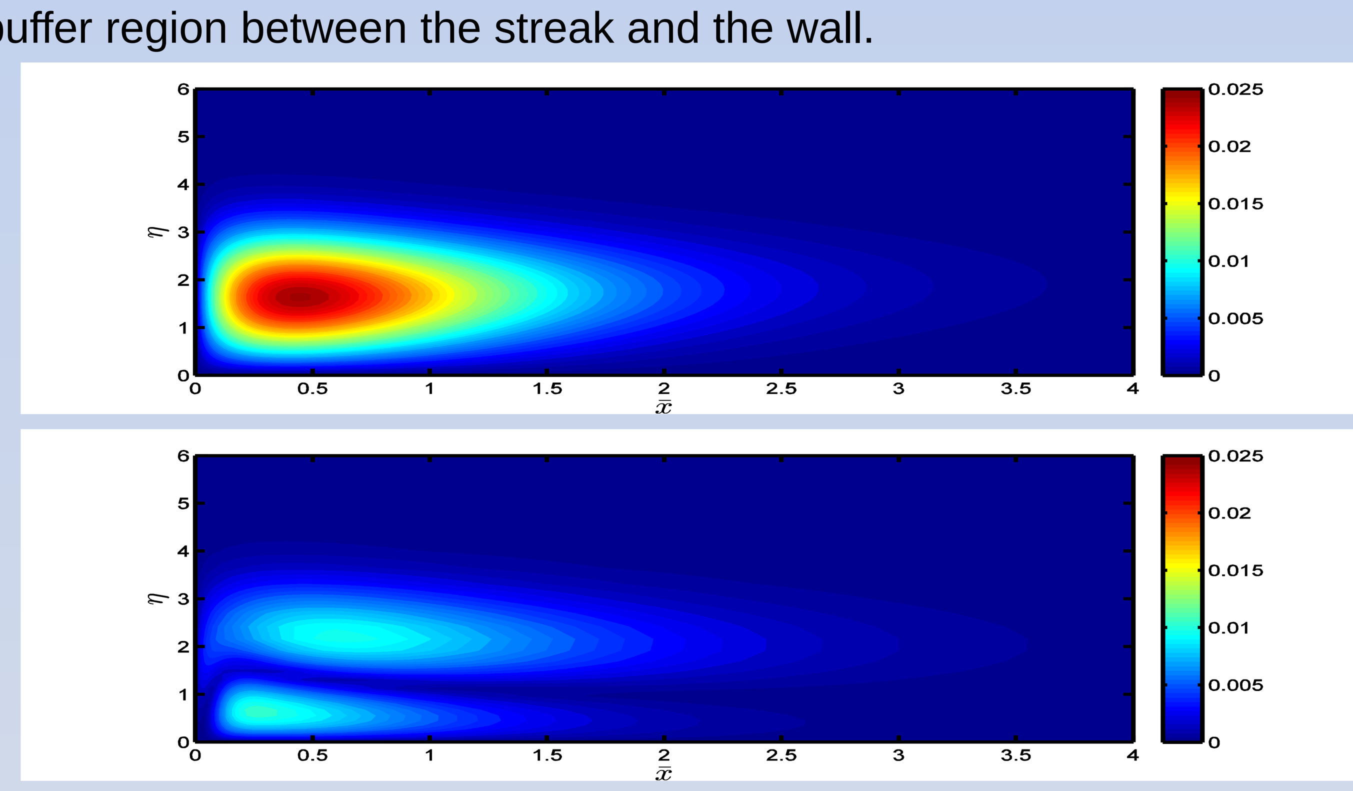

Figure 5: Contour plots of the amplitude of the streamwise perturbation velocity without (left) and with control (right).

In figure 6, the contour plots of the real parts of streamwise vorticity are shown. It is very clear that the streamwise vorticity generated by the free-stream excitation penetrates inside the boundary layer from the top. When the controller is employed at the wall, the streamwise vorticity changes in the near wall area, however the vorticity field far away the wall does not be affected by the controller.

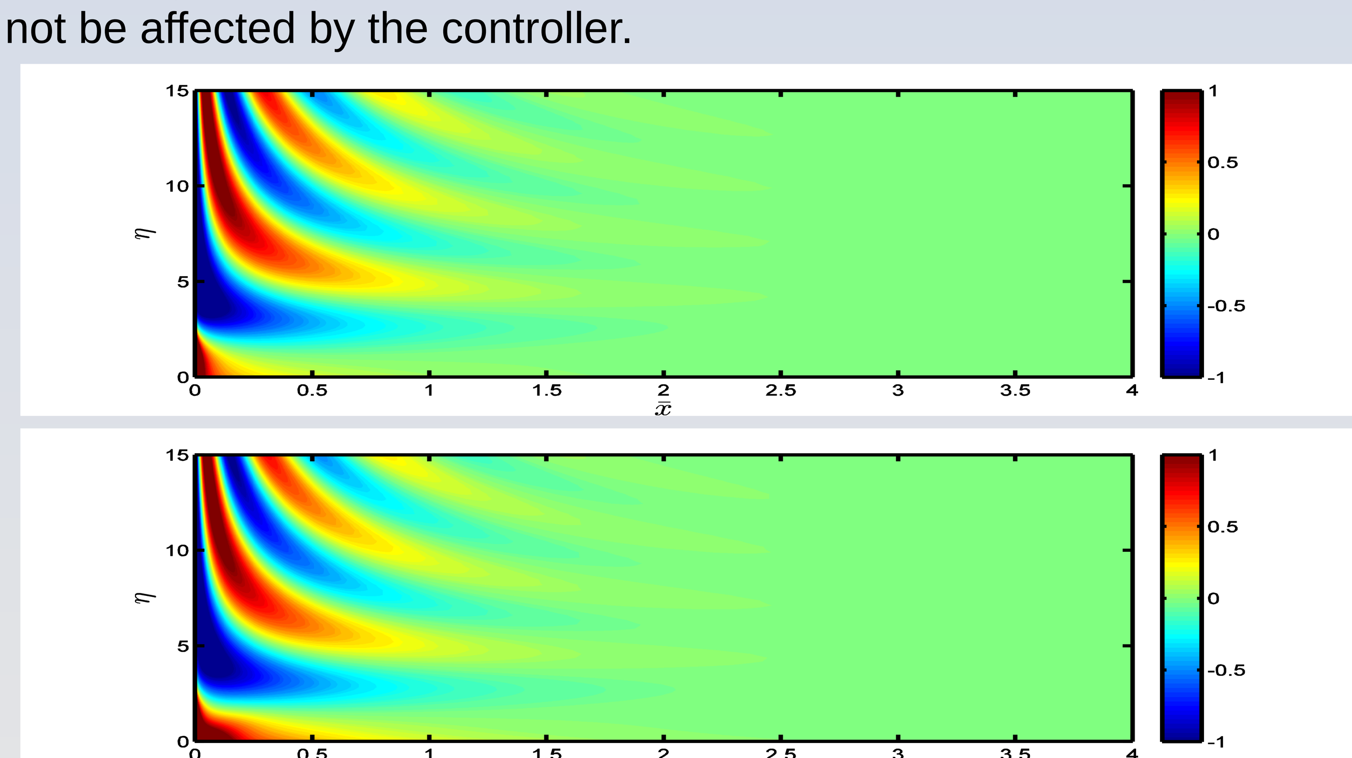

Figure 6: Contour plots of real parts of the streamwise vorticity without (left) and with control (right).

References

S. J. Leib, D. W. Wundrow, and M. E. Goldstein. Effect of free-stream turbulence and other vortical disturbances on laminar boundary layer. J. Fluid Mech., 380:169–203, 1999.

Acknowledgement

This work is supported by the EPSRC grant EP/I016015/1.